Introduction to Computer Science I.
Repeated Second Midterm Test
2016. December 5.

1. Determine the rank of the matrix below for all the values of the parameter x.
$$\begin{pmatrix} 1 & 1 & x \\ 1 & x & x \\ x & x & x \end{pmatrix}$$

2. Let the matrix of the linear mapping $f : \mathbf{R}^3 \to \mathbf{R}^4$ be as below. Determine a linear mapping $h : \mathbf{R}^3 \to \mathbf{R}^4$ for which there is no linear mapping $g : \mathbf{R}^3 \to \mathbf{R}^3$ such that $f \cdot g = h$ holds (and prove your statement).
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

3. Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation which maps each vector to its reflection through the $x = y$ axis. Determine the matrix of f in the basis $B = \{\underline{b}_1 = (3, 7)^T, \underline{b}_2 = (2, 5)^T\}$.

4. We know that the linear transformation $f : \mathbf{R}^2 \to \mathbf{R}^2$ maps the vector $(x, 2)^T$ to $(4, x)^T$ for each $x \in \mathbf{R}$. Determine the eigenvalues and eigenvectors of f.

5. Show that a 2×2 matrix cannot have 3 different eigenvalues.

6. Determine all the integer solutions of the equation $34x + 86y = 4$.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Calculators (or other devices) are not allowed to use.