Introduction to Computer Science I.
First Midterm Test
2016. October 20.

1. We know that the line e perpendicularly intersects the plane of equation $x+2y+3y=6$ at the point $(1,1,1)$; moreover, that the line f contains both the points $(5,2,-1)$ and $(13,4,-5)$. Decide whether e and f have a common point or not.

2. Let $\underline{a}, \underline{b}, \underline{c}$ be linearly independent vectors in $\mathbf{R}^n$. Is it true that in this case the vectors $\underline{a}+\underline{b}+\underline{c}$, $\underline{a}+\underline{b}+3\underline{c}$, $3\underline{a}+\underline{b}+\underline{c}$ are linearly independent as well?

3. Let the subspace V of $\mathbf{R}^4$ consist of those column vectors $\underline{x} \in \mathbf{R}^4$ for which $x_1 = x_2$ and $x_3 = 3x_4$ holds (where x_i denotes the ith coordinate of $\underline{x}$). Determine a basis in the subspace V and show that it is really a basis. (For the solution you don't need to show that V is in fact a subspace.)

4. Determine for which values of the parameter p the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{array}{rcr} x_1 + 2x_2 + x_3 + 3x_4 & = & 2 \\ -x_1 - 2x_2 - x_3 + x_4 & = & -2 \\ 2x_1 + 3x_2 + 4x_3 + 6x_4 & = & 3 \\ 3x_1 + 6x_2 + p \cdot x_3 + 9x_4 & = & p \end{array}$$

5. Is there an integer n for which the value of the determinant below is 0?

$$\begin{vmatrix} 1241 & 1526 & 1566 & n \\ 1914 & 1703 & 896 & 1944 \\ 1552 & 1848 & 1867 & 1956 \\ n+955 & 1896 & 1990 & 1849 \end{vmatrix}$$

6. Are the two 2×2 matrices, A and B for which $A\cdot A = B\cdot B$, but $A \neq B$ and $A \neq -B$? (If the answer is no, prove it; if yes, give an example.)

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.