Introduction to Computer Science I.
Second Midterm Test
2015. December 1.

1. Determine the inverse of the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 3 & -2 \end{pmatrix}$$

2. For the 6×9 matrix A $r(A) = 4$ holds. Show that we can delete one row and one column of A, so that for the remaining 5×8 matrix B $r(B) = 4$ holds. (r denotes the rank of the matrix.)

3. Let A be a 6×9 matrix. We know that there are 5 linearly independent vectors, $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_5 \in \mathbf{R}^3$ for which $A \cdot \underline{v}_1 = \underline{0}$, $A \cdot \underline{v}_2 = \underline{0}, \ldots, A \cdot \underline{v}_5 = \underline{0}$ holds. Show that in this case $r(A) \leq 4$.

4. The matrix of the linear transformation $f : \mathbf{R}^3 \to \mathbf{R}^3$ in the basis $B = \{\underline{b}_1 = (1, 0, 2)^T, \underline{b}_2 = (-2, -1, 0)^T, \underline{b}_3 = (0, -1, 3)^T\}$ is given below. Which element of $\mathbf{R}^3$ is the image of the vector $\underline{b}_1 + 3\underline{b}_2 - 2\underline{b}_3$ under f?

$$[f]_B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

5. Determine the value of the parameter p if we know that 3 is an eigenvalue of the matrix A below. Determine all the eigenvalues and eigenvectors of the matrix A as well.

$$A = \begin{pmatrix} 2 & p \\ 1 & 5 \end{pmatrix}$$

6. What can the remainder of an integer n when divided by 166 be if $71n + 21$ and $33 - 29n$ give the same remainder when divided by 166?

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.