Introduction to Computer Science I.
First Midterm Test
2015. October 27.

1. Determine the point(s) P on the line e with equation $x-6=\frac{y-3}{4}=\frac{1-z}{3}$ for which the line connecting P with $Q(2,-6,5)$ is perpendicular to e.

2. Let $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_k, \underline{w} \in \mathbf{R}^n$ be arbitrary vectors. Suppose that $\underline{w} \neq \underline{0}$ and the set of vectors $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_{i-1}, \underline{v}_i + \lambda \cdot \underline{w}, \underline{v}_{i+1}, \ldots, \underline{v}_k$ is linearly independent for all the choices of the scalar $\lambda \in \mathbf{R}$ and the index $1 \leq i \leq k$. Is it true then that the set $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_k, \underline{w}$ is also linearly independent?

3. Let the subspace V of $\mathbf{R}^4$ consist of those column vectors $\underline{x} \in \mathbf{R}^4$ for which $x_1 - x_2 + x_3 = 0$ and $2x_1 + 3x_2 - x_3 + x_4 = 0$ holds. Determine the dimension of the subspace V. (Here $\underline{x} = (x_1, x_2, x_3, x_4)^T$, as usual. For the solution you don't need to show that V is in fact a subspace.)

4. Determine for which values of the parameter p the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{aligned} x_1 + x_3 + 3x_4 + 8x_5 &= 4 \\ 2x_1 + 3x_2 + 8x_3 + 6x_4 + 10x_5 &= 17 \\ 3x_1 + 2x_2 + 7x_3 + 12x_4 + 8x_5 &= 21 \\ 2x_1 + 4x_2 + 10x_3 + 8x_4 + p \cdot x_5 &= 22 \end{aligned}$$

5. Evaluate the determinant below for all the values of the parameter p (you can use any method).

$$\begin{vmatrix} p & 2 & 3 & p \\ 5 & p & 0 & 0 \\ p & 0 & 4 & p \\ 8 & 5 & p & 8 \end{vmatrix}$$

6. a) Compute the matrix A^{2015} for the matrix A below.
   b) Evaluate the determinant of B^{2015} for the matrix B below.

$$A = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 6 & 5 \\ 1 & 2 \end{pmatrix}$$

   (A^{2015} is the product with 2015 factors, each of whose is A.)

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.