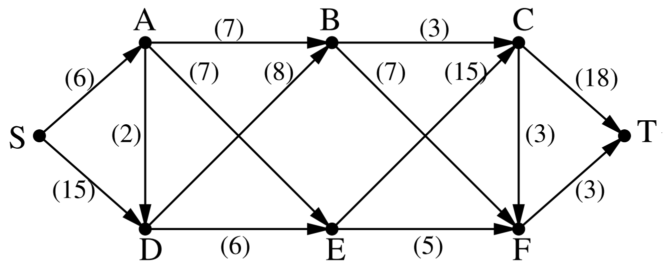
Second Midterm Test

1. In the simple graph G there is a subset X of the vertices which is both an independent set and a covering set. Prove that in this case G is a bipartite graph.

2. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_{15}\}$ and $B = \{b_1, b_2, \ldots, b_{15}\}$. For each $1 \leq i \leq 15$ and $1 \leq j \leq 15$ let a_i and b_j be adjacent if $i \cdot j$ is even. Determine $\nu(G)$, the maximum number of independent edges, $\rho(G)$, the minimum number of covering edges, and give a maximum matching and a minimum covering set of edges in G.

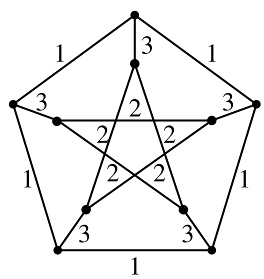
3. The graph G on 15 vertices is constructed from two (vertex-disjoint) cycles on 7 and 8 vertices, respectively, in such a way that we connect each vertex of one cycle with every vertex of the other cycle. Determine $\chi_e(G)$, the edge-chromatic number of the graph G.

4. Determine a maximum flow from S to T and a minimum cut in the network below.



5. Show that if G is a simple, undirected, 12-edge-connected graph on 20 vertices, then its complement cannot be 8-edge-connected.

6. Determine a minimum weight spanning tree in the weighted graph below. How many such trees are there?



Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.