Repeat of the Second Midterm Test

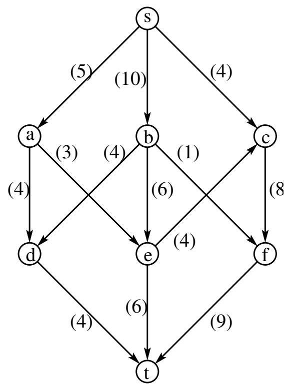
1. Consider all the closed intervals on the real line with both endpoints in the set $\{1, 2, \dots, 6\}$. Determine the chromatic number of the corresponding interval graph.

2. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \dots, a_7\}$ and $B = \{b_1, b_2, \dots, b_8\}$. For each $1 \le i, j \le 7$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine whether G contains a matching covering A or not.

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

3. The graph G on 14 vertices is constructed from two (vertex-disjoint) cycles on 6 and 8 vertices, respectively, in such a way that we connect each vertex of one cycle with every vertex of the other cycle. Determine $\chi_e(G)$, the edge-chromatic number of the graph G.

4. Determine a maximum flow in the network below (from S to T).



5. We we obtain the graph G by deleting the edges of a perfect matching from a complete graph on 10 vertices. Determine the vertex- and edge connectivity numbers ($\kappa(G)$ and $\lambda(G)$) for G.

6. Is it possible that we obtain the BFS spanning trees below started from two different vertices of the graph G?

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.