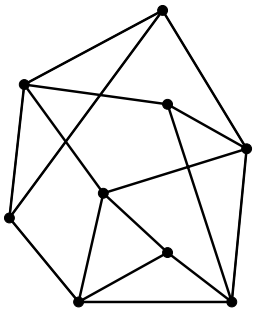
First Midterm Test

1. We want to choose a password for our e-mail account consisting of 8 characters, 3 of which are (not necessarily different) digits (from 0 to 9), and the remaining 5 characters are different letters (from the 26 letters of the English alphabet), such that exactly two of them are uppercase. How many passwords can we choose with these conditions?

2. In a tree there are no vertices of degree 2 or 3. Prove that at least two-thirds of all the vertices have degree 1.

3. Decide whether the following graph is planar or not:



4. Determine the length of the longest trail in K_{10}, the complete graph on 10 vertices.

5. Can we place the numbers $1, 2, \ldots, 15$ on a circle in such a way that the difference of he neighboring numbers in absolute value is at least 4 and at most 7?

6. We delete the edges of two cycles without common vertices (one of length 3 and one of length 4) from the complete graph on 10 vertices. Determine $\chi(G)$, the chromatic number of the graph G obtained this way.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.