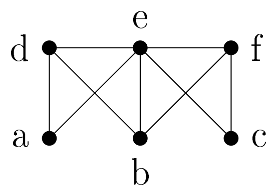
Second Midterm Test

1. Determine whether the following graph is an interval graph or not.

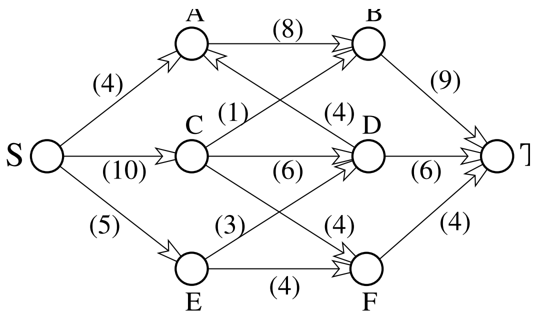


2. Let G be a connected graph on 100 vertices for which $\rho(G) = 99$ ($\rho(G)$ is the minimum number of covering edges in G). Prove that G doesn't contain a cycle of length greater than 3.

3. Let the vertices of the graph G be $u_1, u_2, \ldots, u_{12}$ and $v_1, v_2, \ldots, v_{12}$. Let the vertices u_i and v_j be adjacent if and only if $i \cdot j \leq 12$. There is no other edge in G. Determine $\nu(G)$, the maximum number of independent edges and $\tau(G)$, the minimum number of covering vertices in G.

4. We substitute each edge of K_4, the complete graph on 4 vertices by 2 parallel edges, then subdivide one of the edges of the graph obtained. Determine $\chi_e(G)$, the edge-chromatic number of the final graph G (with 5 vertices and 13 edges). (The subdivision of an edge means that we substitute the edge $\{u, v\}$ by the edges $\{u, x\}$ and $\{x, v\}$, where x is a new vertex.)

5. Determine a maximum flow in the network below (from S to T).



6. The graph G has 2018 vertices and 5000 edges. Show that $\kappa(G) \leq 4$ holds for its vertex-connectivity number.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.