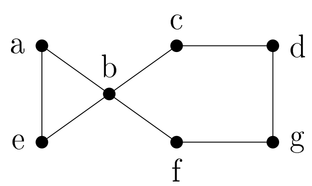
Repeat of the Second Midterm Test

1. Decide whether the following graph is an interval graph or not.

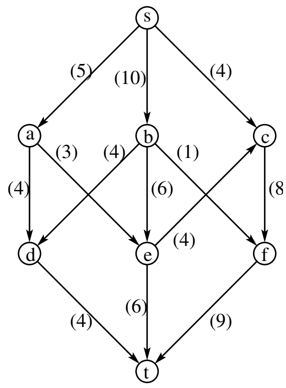


2. Let the vertices of the graph G be $u_1, u_2, \ldots, u_5$ and $v_1, v_2, \ldots, v_5$. Let u_i and u_j be adjacent if and only if $|i-j| = 1$ or 4, v_i and v_j be adjacent also if and only if $|i-j| = 1$ or 4, and u_i and v_j be adjacent for all pairs (i,j) except for $i = j = 1$. (So G consists of two 5-cycles with all the edges but one between them.) Determine $\tau(G)$, the minimum number of covering vertices in G.

3. Let $G = (A, B; E)$ be a bipartite graph. Suppose that for each $X \subseteq A$ for which $|X|$ is odd, $|N(X)| \geq |X| + 1$ holds. Prove that G contains a matching covering A. ($N(X)$ is the set of neighbors of the vertices in X.)

4. In the simple graph G the degree of each vertex is 5 except for one vertex, whose degree is 1. Prove that $\chi_e(G) = 6$ holds for the edge-chromatic number of G.

5. Determine a maximum flow in the network below (from S to T).



6. Determine the maximum number of edges over all simple graphs G for which $\kappa(G) = 1$ holds. ($\kappa(G)$ is the vertex-connectivity number of G.)

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.