

First Midterm Test

1. How many sequences of length 99 can be made using the letters a, b, c, d, e which contain neither adjacent vowels nor adjacent consonants?
2. The simple graph G has 8 edges and the degree of each of its vertices is either 3 or 4. How many vertices can G have? Draw such a graph.
3. Determine the maximum and the minimum of $r(G)$ over all connected simple plane graphs with 8 edges. ($r(G)$ denotes the number of regions.)
4. Let n be an even integer. The graph G on $2n$ vertices is the union of two complete graphs on n vertices each. At least how many edges must be added to G so that the graph obtained is simple and contains an Euler circuit?
5. At least how many edges must be added to the complete bipartite graph $K_{5,6}$ so that the graph obtained contains a Hamilton cycle?
6. Let the vertices of the graph G be u_1, u_2, \dots, u_5 and v_1, v_2, \dots, v_5 . Let u_i and u_j be adjacent if and only if $|i - j| = 1$ or 4 , v_i and v_j be adjacent also if and only if $|i - j| = 1$ or 4 , and u_i and v_j be adjacent for all pairs (i, j) except for $i = j = 1$. (So G consists of two 5-cycles with all the edges but one between them.) Determine $\chi(G)$, the chromatic number of G .

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.

Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.