Repeat of the First Midterm Test

1. How many Hamilton cycles are there in the complete bipartite graph $K_{4,4}$? (Two Hamilton cycles are different if their edge-sets are not the same.)
2. How many vertices does the tree have in which from the two possible degrees one is 10, and it has exactly 10 vertices of degree 10?
3. For the simple plane graph on 100 vertices $e(G) = r(G) + 1$ holds. Determine the possible values of $e(G)$. ($e(G)$ and $r(G)$ denote the number of edges and regions of G.)
4. Prove that no matter how we add edges to the complete bipartite graph $K_{5,5}$ in such a way that the resulting graph G' is simple, G' cannot contain an Euler circuit.
5. Let G be a simple graph on $n \geq 3$ vertices. In $\overline{G}$, the complement of G, the sum of the degrees of any two vertices is at most $n - 2$. Show that G contains a Hamilton path.
6. Let the vertices of the graph G be $v_1, v_2, \ldots, v_{100}$, and let v_i and v_j be adjacent if and only if $|i - j| = 1, 4$ or 5. Determine $\chi(G)$, the chromatic number of G.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.