Second Repeat of the First Midterm Test

1. How many seven-digit telephone numbers are there whose first digit is one of the numbers 1, 2, 3, 4, and the first digit is repeated at least once more in the number?

2. In a simple graph on 23 vertices the degree of each vertex is at least 7. Show that no matter how we choose three vertices of the graph, there will be a path between two of them.

3. Determine the maximum and the minimum of $r(G)$ over all simple (not necessarily connected) plane graphs with 6 edges. ($r(G)$ denotes the number of regions.)

4. At most how many edges can be added to $K_{4,4}$ so that the graph obtained is simple and contains an Euler circuit?

5. Let G be a graph on n vertices, where $n \geq 6$. In $\overline{G}$, the complement of G, no three vertices have a common neighbor. Show that G contains a Hamilton cycle.

6. Let the vertex set of the graph G on 9 vertices be the vertices of the unit cube together with the center of it, i.e. $V(G) = \{(x, y, z) : x, y, z \text{ are 0 or 1}\} \cup \{(1/2, 1/2, 1/2)\}$. Let two vertices of G be adjacent if they differ either in the first or the second coordinate, or both. (E.g. $(0, 0, 1)$ is adjacent to $(0, 1, 1)$ and $(1, 1, 0)$ but not to $(0, 0, 0)$.) Determine $\chi(G)$, the chromatic number of G.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.