Second Midterm Test

1. The simple graph G contains exactly one odd cycle. Determine the chromatic number of G.

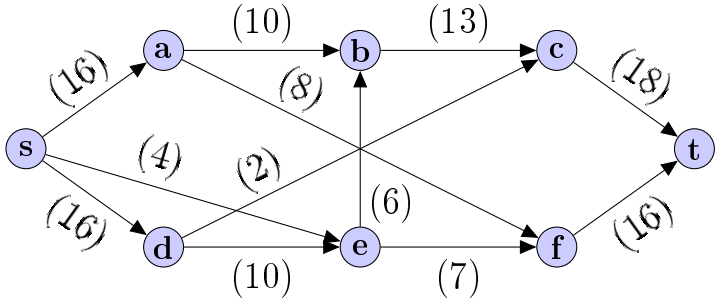
2. We delete the edges of a cycle on 6 vertices from a complete graph on 8 vertices. Determine the chromatic number of the graph obtained.

3. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_8\}$ and $B = \{b_1, b_2, \ldots, b_8\}$. For each $1 \leq i \leq 8$ and $1 \leq j \leq 8$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine a maximum matching and a minimum covering set in G.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

4. In a ball for informaticians there are 12 girls and 198 boys. The organizers thus want to form 12 couples for the opening dance in such a way that everybody dances with somebody he/she knows. Every girl knows at least 11 boys but each boy knows at most 11 girls (acquaintances are mutual). Can the organizers form the 12 couples?

5. Determine a maximum flow and a minimum s, t-cut in the network below.



6. A simple graph on 10 vertices has 40 edges. Determine the largest integer k for which G is surely k-vertex-connected.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.