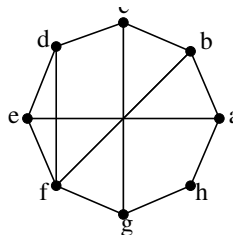
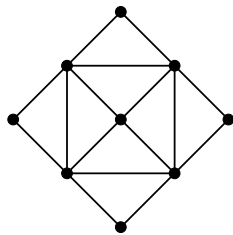


1. In an imaginary country the license plate numbers consist of 6 characters, each of which is either one of the 26 letters of the English alphabet or one of the digits $0, 1, \dots, 9$. Three characters must be letters and three must be numbers, with the condition that if three letters stand next to each other then all of them cannot be the same (e.g. a good license plate number is 37AAG1, but ABCD85 and 35HHH2 are not appropriate). How many license plate numbers can be given in the country?
2. A simple connected graph on 100 vertices has 102 edges. Show that the graph contains three pairwise different cycles. (Two cycles are different if their edge sets are not the same.)
3. Let G be a connected graph on 100 vertices. We assigned weights 1 or 2 to the edges of G in such a way that the subgraph consisting of the edges of weight 1 has 7 components. (The subgraph consisting of the edges of weight 1 contains all the vertices of G and the edges of G of weight 1.) Determine the weight of a minimum weight spanning tree in G .
4. Decide whether the following graph is planar or not.



5. Does the following graph contain a Hamilton cycle? And a Hamilton path?



6. In a simple graph on 20 vertices the degree of each vertex is 8. Prove that we can add 20 new edges to the graph in such a way that the resulting graph is still simple and contains an Euler circuit.

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.

Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.