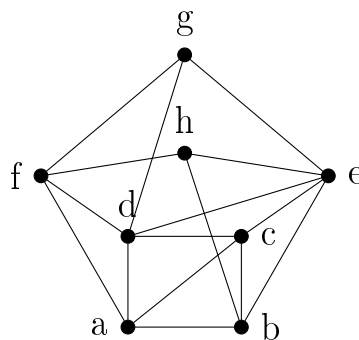
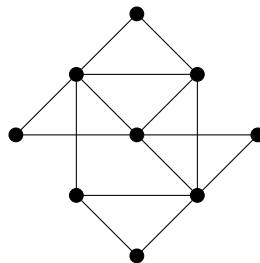


Second Repeat of the First Midterm Test

1. How many seven-digit integers are there in which the digit 8 occurs exactly three times?
2. In a simple graph on 23 vertices the degree of each vertex is at least 7. Show that no matter how we choose 3 vertices of the graph, there will be a path between 2 of them.
3. In a simple, connected weighted graph on 10 vertices three edges have weight 1, four edges have weight 2, and the rest of the edges have weight 3. Show that the graph has a spanning tree whose weight is at most 21.
4. Decide whether the following graph is planar or not.



5. At least how many edges need to be added to the graph below so that the resulting graph is still simple and contains an Euler trail?



6. In a simple graph on 20 vertices and its complement together there are only two kinds of degrees. Furthermore we know that in the graph the vertices of degree less than 10 form a clique. Show that the graph contains a Hamilton path.

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Show all your

work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.

Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.