1. Let the vertices of the graph $G$ be the all the 0-1 sequences of length 5, and two sequences be adjacent if they differ in exactly one position. Is the graph $G$ a bipartite graph?

2. Let the vertex set of the graph $G$ be $V(G) = \{1, 2, \ldots, 30\}$. Let the vertices $x, y \in V(G)$ be adjacent in $G$ if the difference of the numbers $x$ and $y$ is at least 7. Determine $\chi(G)$, the chromatic number of $G$.

3. Let the two vertex classes of the bipartite graph $G(A,B; E)$ be $A = \{a_1, a_2, \ldots, a_8\}$ and $B = \{b_1, b_2, \ldots, b_8\}$. For each $1 \leq i, j \leq 8$ let $a_i$ and $b_j$ be adjacent if the entry in the $i$th row and $j$th column of the matrix below is 1. Determine $\nu(G)$, the maximum number of independent edges, $\rho(G)$, the minimum number of covering edges, and give a maximum matching and a minimum covering set of edges in $G$.

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 1
\end{pmatrix}
\]

4. The chromatic number of the simple graph $G$ is $\chi(G) = 3$ and there is a coloring of the vertices of $G$ with 3 colors in which one of the colors appears on one vertex only. Show that $\tau(G) \leq \nu(G) + 1$ holds for $G$, where $\tau(G)$ is the minimum number of covering vertices and $\nu(G)$ is the maximum number of independent edges in $G$.

5. Let the vertex set of the graph $G$ be $V(G) = \{1, 2, \ldots, 30\}$. Let the vertices $x, y \in V(G)$ be adjacent in $G$ if $x \neq y$ and $x \cdot y$ is divisible by 7. Determine $\chi_e(G)$, the edge-chromatic number of $G$.

6. Determine a maximum flow and a minimum cut in the network below.