Second Repeat of the Second Midterm Test

1. In a simple graph G on 10 vertices the degree of each vertex is 8. Determine the chromatic number of G.

2. In a simple graph G on 10 vertices the maximum degree is 6, and the maximum number of independent edges is 5. Show that G contains an odd cycle.

3. In a simple bipartite graph G on 9 vertices the degree of each vertex is either 2 or 4. Show that G contains a matching of 3 edges.

4. A simple graph G on 10 vertices contains one vertex of degree 5, one of degree 4, one of degree 3, and the rest of the vertices have degree 2. Show that G can be colored with 3 colors.

5. Show that if G is a simple k-regular graph on 9 vertices then $\chi_e(G) + \chi_e(\overline{G}) \geq 10$.

6. Determine a maximum flow and a minimum cut in the network below.
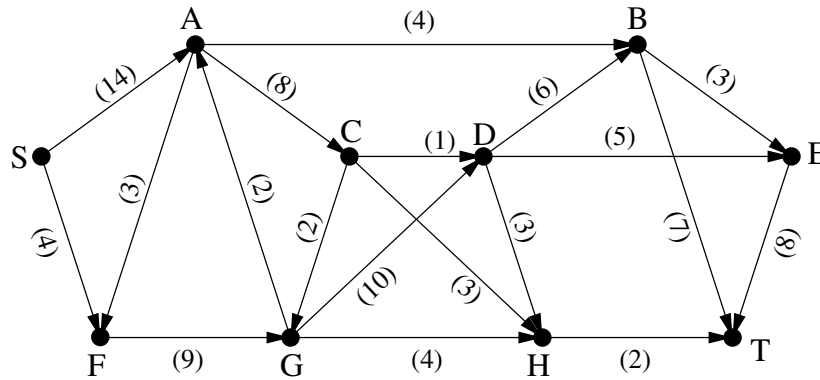
Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.