1. How many sequences of letters can be made using the 26 letters of the English alphabet which contain exactly 4 X’s and (exactly) 3 Y’s?

2. In the simple graph $G$ on 20 vertices 10 vertices have degree at most 7, and the other 10 vertices have degree at least 16. How many edges are there in $G$?

3. The BFS algorithm visited the vertices of the graph below in the following order: S, □, □, □, H, □, F, C, □. Complete the sequence with the missing vertices (which are denoted by □), and determine the corresponding BFS tree.

   a) Can the edge \{D, H\} be contained in an arbitrary BFS spanning tree started from S?

   b) Can the edge \{D, H\} be contained in an arbitrary BFS spanning tree started from S?

   ![Graph Image]

4. Let $G$ be a connected graph and $w : E(G) \rightarrow \mathbf{R}$ be a weight function on the edges of $G$. Suppose that one of the endpoints of the edge $e$ of $G$ is $v$ and for all the edges $f$ which are incident to $v$ the inequality $w(e) \leq w(f)$ holds. Show that $G$ has a minimum weight spanning tree which contains $e$.

5. At most how many edges can be added to the graph below in such a way that we get a simple planar graph? (We add edges only between already existing vertices.)

   ![Graph Image]

6. The graph $G$ is a star on 101 vertices (i.e. $G$ has one vertex of degree 100 and hundred vertices of degree 1). At least how many edges must be added to $G$ so that the graph obtained contains a Hamilton cycle?

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points.