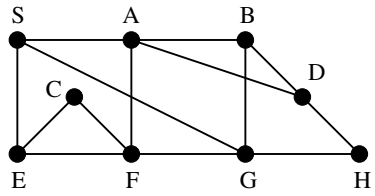
1. How many sequences of letters can be made using the 26 letters of the English alphabet which contain exactly 4 X's and (exactly) 3 Y's?

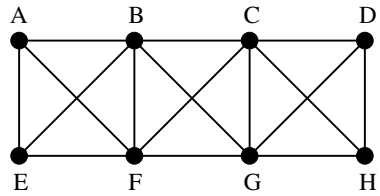
2. In the simple graph G on 20 vertices 10 vertices have degree at most 7, and the other 10 vertices have degree at least 16. How many edges are there in G?

3. The BFS algorithm visited the vertices of the graph below in the following order: $S, \square, \square, \square, H, \square, F, C, \square$. Complete the sequence with the missing vertices (which are denoted by $\square$), and determine the corresponding BFS tree.
   b) Can the edge $\{D, H\}$ be contained in an arbitrary BFS spanning tree started from S?



4. Let G be a connected graph and $w : E(G) \to \mathbf{R}$ be a weight function on the edges of G. Suppose that one of the endpoints of the edge e of G is v and for all the edges f which are incident to v the inequality $w(e) \leq w(f)$ holds. Show that G has a minimum weight spanning tree which contains e.

5. At most how many edges can be added to the graph below in such a way that we get a simple planar graph? (We add edges only between already existing vertices.)



6. The graph G is a star on 101 vertices (i.e. G has one vertex of degree 100 and hundred vertices of degree 1). At least how many edges must be added to G so that the graph obtained contains a Hamilton cycle?

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.