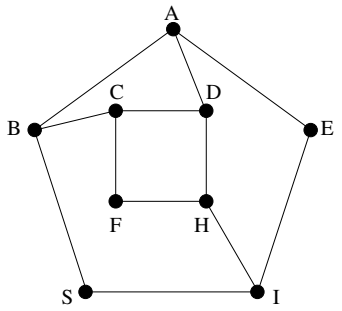
Introduction to Computer Science

Repeated First Midterm Test, 5/4/2015

1. How many 5-element subsets does the set $\{1, 2, \ldots, 10\}$ have which contain more even numbers than odd numbers?

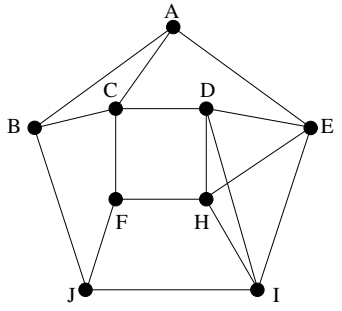
2. Does there exist a simple graph on 21 vertices for which it holds that both G and its complement $\overline{G}$ contain 9 vertices of degree 4 and 3 vertices of degree 10?

3. From the graph below one edge is missing (but all the vertices are there). Can we determine for sure which two vertices were connected by the missing edge if we know that the BFS algorithm started from the vertex S visited the vertices of the graph in the following order:
   a) $S, B, I, C, A, F, H, E, D$;
   b) $S, I, B, E, F, H, C, A, D$?
   In the case where the missing edge can be determined unambiguously, determine the corresponding BFS-tree as well.



4. Let G be a connected graph and $w : E(G) \to \mathbf{R}$ be a weight function on the edges of G. Furthermore, let C be a cycle in G and e an edge of C. Suppose that $w(e) \leq w(f)$ holds for all the edges f of the cycle C. Show that G has a minimum weight spanning tree which doesn't contain e.

5. Decide whether the following graph is planar or not. If yes, then draw it with straight edges without crossing, if not, then prove it.



6. Let G be a simple connected graph on 30 vertices. We color 10 of its vertices red, 10 vertices blue and 10 vertices green in such a way that every vertex is adjacent to at least 5 vertices of the same color. Show that G contains a path of length at least 25.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.