

List of Questions

1. Enumeration: permutations, variations, combinations (without and with repetition). Simple relations between binomial coefficients, binomial theorem, Pascal's triangle.
2. Basic notions of graph theory: graph, simple graph, degree, isomorphism, complement, subgraph, walk, trail, circuit, path, cycle, connectedness, components. Trees: basic properties, spanning trees, their existence.
3. Planar (and plane) graphs: connection with drawing graphs on the sphere. Euler's theorem and its consequences, Kuratowski graphs, Kuratowski's theorem (partial proof). Fary-Wagner theorem (no proof). Duality, its properties, connections between the original and dual graphs.
4. Euler trail and circuit, necessary and sufficient conditions for their existence. Hamilton path and cycle, necessary conditions, sufficient conditions: Dirac's and Ore's theorem.
5. Vertex coloring: the notion of $\chi(G)$ and its relationship to $\omega(G)$ and $\Delta(G)$. Mycielski's construction. Greedy coloring. Chromatic number of planar graphs. Interval graphs, their coloring.
6. Bipartite graphs, relationship with odd cycles. Covering and independent vertices and edges. Galai's theorems (partial proofs). Tutte's theorem (partial proof).
7. Matchings. Augmenting paths. Theorems of König, Hall and Frobenius. Edge-chromatic number, its relationship to $\Delta(G)$ and $\nu(G)$. Vizing's theorem (no proof), Shannon's theorem (no proof). König's theorem (edge-chromatic number of bipartite graphs).
8. Network, flow, value of a flow, s-t cut, capacity of a cut, augmenting paths. Ford-Fulkerson theorem, Edmonds-Karp theorem (no proof). Integrality lemma.
9. Generalizations of flows. Menger's theorems about paths between pairs of points (partial proofs). Higher connectivity and edge-connectivity in graphs. Menger's related theorems.
10. BFS algorithm, its usage for determining connectedness and distances. Minimum weight spanning tree, Kruskal's theorem (no proof).
11. Algorithms for finding shortest paths: Dijkstra's algorithm, Ford's algorithm, Floyd's algorithm.
12. DFS algorithm, DFS tree, classification of the edges. DAG, topological ordering. Shortest and longest paths in acyclic graphs.