

Exercise-set 7. Solutions

1. a) Vertices of the same color are independent.
b) It is a good coloring, if each vertex in the complement of a maximum independent set of vertices gets a different color.
2. No (counterexample).
3. a) It is an interval graph (\exists a representation of the vertices with intervals).
b) It is not an interval graph, since $\chi(G) \neq \omega(G)$.
c) It is not an interval graph, since there is no representation of the vertices with intervals.
4. It is an interval graph (\exists a representation of the vertices with intervals).
5. It is an interval graph (\exists a representation of the vertices with intervals).
6. $\chi(G) = \omega(G) = 11$ (=max. # of intervals through a point).
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8. $\omega = 10$ in the original graph. We can delete at most 2 vertices from a clique $\implies \chi = \omega \geq 8$.
9. $\chi(G) = 3$, $\nu(G) = 9$, $\tau(G) = 12$, $\alpha(G) = 6$, $\rho(G) = 9$.
10. a) $\nu(G) = 4$, $\tau(G) = 4$, $\alpha(G) = 6$, $\rho(G) = 6$.
b) $\nu(G) = 5$, $\tau(G) = 5$, $\alpha(G) = 7$, $\rho(G) = 7$.
a) $\nu(G) = 4$, $\tau(G) = 4$, $\alpha(G) = 6$, $\rho(G) = 6$.
11. $G = K_{668} \cup K_{668,669} \implies \chi(G) = 668$, $\nu(G) = 334 + 668 = 1002$, $\tau(G) = 667 + 668 = 1335$, $\alpha(G) = 1 + 669 = 670$, $\rho(G) = 334 + 669 = 1003$.
12. $\nu(G) = 20 = \tau(G)$.
13. $\alpha(G) = 86$, $\tau(G) = 14$, $\nu(G) = 14$, $\rho(G) = 86$.
14. $\nu(G) = 25$, $\alpha(G) = 75$.
15. $\nu(G) = \tau(G) = \rho = \alpha(G) = 8$ (actually it is a bipartite graph).
16. $\nu(G) = 4 = \tau(G)$ (actually it is a bipartite graph).
17. a) By contradiction: otherwise the matching would not be maximum.
b) Follows from a).
c) Follows from b) and Gallai's theorem.
18. X and \bar{X} are both independent.
19. a) True.
b) False.
c) No.
20. G contains a Hamilton cycle $\implies \nu(G) \geq \lfloor 2k + 1/2 \rfloor = k$, and $\nu(G) \leq (2k + 1)/2 = k$.
21. Yes (the graph contains a Hamilton cycle).
22. If we add the edge $\{u, v\}$ to G then it contains a Hamilton cycle.
23. If we add two new vertices (connected to all the old ones) to G then the new graph contains a Hamilton cycle (see set 5, ex. 5.)
24. $\det M \neq 0 \implies \exists$ a nonzero elementary product, corresponding to a perfect matching.
25. $|E(G)| \leq \binom{20}{2} + 20 \cdot 80 = 1790$, and this is possible (example).
26. $\nu(G) = 10 = \tau(G)$.
27. No edge can be missing from G .
28. Count the number of edges incident to a minimum covering set.
29. We can choose the independent vertices greedily, one by one.