Exercise-set 3.
Solutions

1. a) For non-simple graphs no.
   b) For simple graphs yes: they cannot have (at least) 2 components.
2. The graph cannot have 3 components.
3. There are at most 2 components.
4. If G is not connected, then the edges between the components of it make $\overline{G}$ connected.
5. In every connected component of a graph there is an even number of odd-degree vertices.
6. a) $n_1 \cdot 1 + n_2 \cdot 2 + 5 \cdot 3 = 2(n_1 + n_2 + 5 - 1) \implies n_1 = 7$.
7. $2 \cdot 1 + (n-3) \cdot 2 + 1 \cdot d = 2(n-1) \implies d = 2$.
8. One of the degrees is 1. $d \cdot 9 + 92 \cdot 1 = 200 \implies d = 12$.
9. The tree has an even number of vertices.
10. $10(n-1) = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 22$.
11. Necessary: $n - 1 = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 4$. Both are possible.
12. One of the degrees is 1, the other is at least 4. $n_1 \cdot 1 + (n - n_1) \cdot 4 \leq 2(n-1) \implies n_1 \geq \frac{2n+2}{3}$.
13. One of the degrees is 1. $d \cdot n_1 + 1 \cdot (11 - n_1) = 20 \implies n_1(d-1) = 9 \implies n_1 = 1, 3$ or $9 \implies 3$ non-isomorphic trees (draw).
14. e_2 is an edge in the cycle in $T_2 + e_1$ which connects the two components of $T_1 - e_1$.
15. a) No,
    b) yes.
16. A graph is a spanning tree and 3 more edges, each of which forms a cycle with the tree.
17. The graph contains a cycle, of lenght at least 3.
18. The number of edges in a spanning forest is 17.
19. A degree one vertex in a spanning tree is like that.
20. a) yes,
    b) no,
    c) no,
    d) yes.
21. a) $S, G, E, A, H, B, F, C, D$.
    b) No.
22. a) The edge not in the BFS spanning tree started from s whose endpoints are closest to s determines such a cycle, if the first common ancestor of its endpoints is s.
    b) Start a BFS in $G - e$ from one of the endpoints of e.
23. a) no,
    b) yes,
    c) yes.
24. 99 (must be a tree).
25. No (check the distances).
26. Not true.