Combinatorics and Graph Theory 1.

Exercise-set 10.
Solutions

1. a) max $m(f) = 8$, min cut: $X = \{S, A, F\}$,
   b) max $m(f) = 20$, min cut: $X = \{S, A, B, C\}$,
   c) max $m(f) = 30$, min cut: $X = \{S, B, C, E\}$,
   d) max $m(f) = 17$, min cut: $X = \{S, B, C, D, E\}$,
   e) max $m(f) = 24$, min cut: $X = \{S, A, D, G\}$,
   f) max $m(f) = 21$, min cut: $X = \{S, D, F\}$,
   g) max $m(f) = 14$, min cut: $X = \{S, A, B, F, I\}$,
   h) max $m(f) = 24$, min cut: $X = \{S, B, D, E, F\}$.

2. The capacity of the cut is 19, max $m(f) = 18$, min cut: $X = \{S, A, B, G, H\}$.

3.

4. max $m(f) = 18$, min cut: $X = \{S, B, D\}$.

5. max $m(f) = 26$, min cut: $X = \{S, D, F\}$.

6. max $m(f) = 43$, min cut $X = \{s, a, b, c, d\}$.

7. Yes: e must be in the minimum cut.

8. True (we can use augmenting paths of smaller values).

9. The s, t-cut with $X = V \setminus \{t\}$ is a minimum s, t-cut.

10. The min s, w-cut has capacity at least 100.

11. a) True.
    b) True.
    c) False.
    d) We get the same answers as for a), b), c).

12.