

**Exercise-set 7.**  
**Solutions**

1. a) Vertices of the same color are independent.  
b) It is a good coloring, if each vertex in the complement of a maximum independent set of vertices gets a different color.
2. No (counterexample).
3. a) It is an interval graph ( $\exists$  a representation of the vertices with intervals).  
b) It is not an interval graph, since  $\chi(G) \neq \omega(G)$ .  
c) It is not an interval graph, since there is no representation of the vertices with intervals.
4. It is an interval graph ( $\exists$  a representation of the vertices with intervals).
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6.  $\chi(G) = \omega(G) = 11$  (=max. # of intervals through a point).
7.  $\chi(G) = \omega(G) = 11$  (=max. # of intervals through a point).
8.  $\omega = 10$  in the original graph. We can delete at most 2 vertices from a clique  $\implies \chi = \omega \geq 8$ .
9.  $\chi(G) = 3$ ,  $\nu(G) = 9$ ,  $\tau(G) = 12$ ,  $\alpha(G) = 6$ ,  $\rho(G) = 9$ .
10. a)  $\nu(G) = 4$ ,  $\tau(G) = 4$ ,  $\alpha(G) = 6$ ,  $\rho(G) = 6$ .  
b)  $\nu(G) = 5$ ,  $\tau(G) = 5$ ,  $\alpha(G) = 7$ ,  $\rho(G) = 7$ .  
a)  $\nu(G) = 4$ ,  $\tau(G) = 4$ ,  $\alpha(G) = 6$ ,  $\rho(G) = 6$ .
11.  $G = K_{668} \cup K_{668,669} \implies \chi(G) = 668$ ,  $\nu(G) = 334 + 668 = 1002$ ,  $\tau(G) = 667 + 668 = 1335$ ,  $\alpha(G) = 1 + 669 = 670$ ,  $\rho(G) = 334 + 669 = 1003$ .
12.  $\nu(G) = 20 = \tau(G)$ .
13.  $\alpha(G) = 86$ ,  $\tau(G) = 14$ ,  $\nu(G) = 14$ ,  $\rho(G) = 86$ .
14.  $\nu(G) = 25$ ,  $\alpha(G) = 75$ .
15.  $\nu(G) = \tau(G) = \rho = \alpha(G) = 8$ .
16. a) By contradiction: otherwise the matching would not be maximum.  
b) Follows from a).  
c) Follows from b) and Gallai's theorem.
17.  $X$  and  $\bar{X}$  are both independent.
18. a) True.  
b) False.  
c) No.
19.  $G$  contains a Hamilton cycle  $\implies \nu(G) \geq \lfloor 2k + 1/2 \rfloor = k$ , and  $\nu(G) \leq (2k + 1)/2 = k$ .
20. If we add the edge  $\{u, v\}$  to  $G$  then it contains a Hamilton cycle.
21. If we add two new vertices (connected to all the old ones) to  $G$  then the new graph contains a Hamilton cycle.
22.  $\det M \neq 0 \implies \exists$  a nonzero elementary product, corresponding to a perfect matching.
23.  $|E(G)| \leq \binom{20}{2} + 20 \cdot 80 = 1790$ , and this is possible (example).
24.  $\nu(G) = 10 = \tau(G)$ .
25. No edge can be missing from  $G$ .
26. Count the number of edges incident to a minimum covering set.