Exercise-set 3.
Solutions

1. a) $n_1 \cdot 1 + n_2 \cdot 2 + 5 \cdot 3 = 2(n_1 + n_2 + 5 - 1) \implies n_1 = 7$.
2. $2 \cdot 1 + (n-3) \cdot 2 + 1 \cdot d = 2(n-1) \implies d = 2$.
3. One of the degrees is 1. $d \cdot 9 + 92 \cdot 1 = 200 \implies d = 12$.
4. The tree has an even number of vertices.
5. $10(n-1) = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 22$.
6. Necessary: $n - 1 = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 4$. Both are possible.
7.
8. One of the degrees is 1, the other is at least 4. $n_1 \cdot 1 + (n - n_1) \cdot 4 \leq 2(n-1) \implies n_1 \geq \frac{2n+2}{3}$.
9. One of the degrees is 1. $d \cdot n_1 + 1 \cdot (11 - n_1) = 20 \implies n_1(d-1) = 9 \implies n_1 = 1, 3$ or $9 \implies 3$ non-isomorphic trees (draw).
10. a) No,
    b) yes.
11. A graph is a spanning tree and 3 more edges, each of which forms a cycle with the tree.
12. The graph contains a cycle, of lenght at least 3.
13. The number of edges in a spanning forest is 17.
14. A degree one vertex in a spanning tree is like that.
15. a) yes,
    b) no,
    c) no,
    d) yes.
16. a) $S, G, E, A, H, B, F, C, D$.
    b) No.
17. The edge not in the BFS spanning tree started from s whose endpoints are closest to s determines such a cycle.
18. a) no,
    b) yes,
    c) yes.
19.
20. No (check the distances).
21. a) There are 36 minimum weight spanning trees of weight 19.
    b) There are 125 minimum weight spanning trees of weight 15.
22. There are 99! minimum weight spanning trees of weight $2 + 3 + \cdots + 100 = 5049$.
23. The weight of a minimum weight spanning tree is 150.
24. By Kruskal's algorithm: when we get to e, we cannot create a cycle.
25. By Kruskal's algorithm: the other edges of C can be selected before e.
26.
27. 2 cases: $p \leq 2$ and $p = 2$.