Exercise-set 2.
Solutions

1. (a) yes, yes,
   (b) no, no,
   (c) no, yes,
   (d) no, yes.

2. 8 (the smallest possible and also realizable degree-sequence is 0,1,2,3,4,6).

3. There are $n-1$ odd degrees $\implies$ $n-1$ is even, n is odd $\implies$ one even degree in $\overline{G}$.

4. The number of edges between the vertices of degree ≤ 7 and the vertices of degree ≥ 16 is both ≤ 70 and ≥ 70 $\implies = 70$ $\implies$ $|E(G)| = 115$.

5. Count the number of edges between the k vertices of highest degree and the rest in two ways.

6. There are 11 of them.

7. There are 2 of them.

8. a) There are 4 such graphs,
   b) there are 4 such graphs,
   c) there are 2 such graphs.

9. a) Yes, yes, no (should have 15/2 edges).
   b) No (should have 11 vertices and 55/2 edges).

10. 4.

11. a) 6,
    b) Only 1.

12. a) no, b) yes.

13. a) and b) are isomorphic, and c) is not isomorphic to them.

14. a) and b) are isomorphic to it, c) is not.

15. No. The underlying graph is a cycle,...

16. a) For non-simple graphs no.
    b) For simple graphs yes: they cannot have (at least) 2 components.

17. The graph cannot have 3 components.

18. There are at most 2 components.

19. If G is not connected, then the edges between the components of it make $\overline{G}$ connected.

20. In every connected component of a graph there is an even number of odd-degree vertices.