Combinatorics and Graph Theory 1.

## Exercise-set 10. Solutions

- 1. a) max m(f) = 8, min cut:  $X = \{S, A, F\},\$ 
  - b) max m(f) = 20, min cut:  $X = \{S, A, B, C\},\$
  - c) max m(f) = 30, min cut:  $X = \{S, B, C, E\},\$
  - d) max m(f) = 17, min cut:  $X = \{S, B, C, D, E\},\$
  - e) max m(f) = 24, min cut:  $X = \{S, A, D, G\}$ ,
  - f) max m(f) = 21, min cut:  $X = \{S, D, F\},$
  - g) max m(f) = 14, min cut:  $X = \{S, A, B, F, I\}$ , b) max m(f) = 24 min cut:  $X = \{S, B, D, F, F\}$
  - h) max m(f) = 24, min cut:  $X = \{S, B, D, E, F\}$ .
- 2. The capacity of the cut is 19, max m(f) = 18, min cut:  $X = \{S, A, B, G, H\}$ .
- 3. max m(f) = 18, min cut:  $X = \{S, B, D, 3mnjF\}$ .
- 4. max m(f) = 18, min cut:  $X = \{S, B, D\}$ .
- 5. max m(f) = 26, min cut:  $X = \{S, D, F\}$ .
- 6. Yes: e must be in the minimum cut.
- 7. True (we can use augmenting paths of smaller values).
- 8. The s,t-cut with  $X = V \setminus \{t\}$  is a minimum s,t-cut.
- 9. The min s, w-cut has capacity at least 100.
- 10. a) True.
  - b) True.
  - c) False.
  - d) We get the same answers as for a), b), c).
- 11. a) True.
  - b) Not true, if there are two edge-disjoint minimum cuts.
  - c) The edges which are in all the minimum cuts satisfy b). The algorithm for finding them is like in b).