Combinatorics and Graph Theory 1.

Exercise-set 10.

Solutions

1. a) \( \max_m f = 8 \), min cut: \( X = \{ S, A, F \} \),
   
b) \( \max_m f = 20 \), min cut: \( X = \{ S, A, B, C \} \),
   
c) \( \max_m f = 30 \), min cut: \( X = \{ S, B, C, E \} \),
   
d) \( \max_m f = 17 \), min cut: \( X = \{ S, B, C, D, E \} \),
   
e) \( \max_m f = 24 \), min cut: \( X = \{ S, A, D, G \} \),
   
f) \( \max_m f = 21 \), min cut: \( X = \{ S, D, F \} \),
   
g) \( \max_m f = 14 \), min cut: \( X = \{ S, A, B, F, I \} \),
   
h) \( \max_m f = 24 \), min cut: \( X = \{ S, B, D, E, F \} \).

2. The capacity of the cut is 19, \( \max_m f = 18 \), min cut: \( X = \{ S, A, B, G, H \} \).

3. \( \max_m f = 18 \), min cut: \( X = \{ S, B, D, 3mnjF \} \).

4. \( \max_m f = 18 \), min cut: \( X = \{ S, B, D \} \).

5. \( \max_m f = 26 \), min cut: \( X = \{ S, D, F \} \).

6. Yes: e must be in the minimum cut.

7. True (we can use augmenting paths of smaller values).

8. The \( s, t \)-cut with \( X = V \setminus \{ t \} \) is a minimum \( s, t \)-cut.

9. The min \( s, w \)-cut has capacity at least 100.

10. a) True.
    
    b) True.
    
    c) False.
    
    d) We get the same answers as for a), b), c).

11. a) True.
    
    b) Not true, if there are two edge-disjoint minimum cuts.
    
    c) The edges which are in all the minimum cuts satisfy b). The algorithm for finding them is like in b).