Exercise-set 9.
Solutions

1. a) max $m(f) = 8$, min cut: $X = \{S, A, F\}$,
   b) max $m(f) = 20$, min cut: $X = \{S, A, B, C\}$,
   c) max $m(f) = 30$, min cut: $X = \{S, B, C, E\}$,
   d) max $m(f) = 17$, min cut: $X = \{S, B, C, D, E\}$,
   e) max $m(f) = 24$, min cut: $X = \{S, A, D, G\}$,
   f) max $m(f) = 21$, min cut: $X = \{S, D, F\}$,
   g) max $m(f) = 14$, min cut: $X = \{S, A, B, F, I\}$,
   h) max $m(f) = 24$, min cut: $X = \{S, B, D, E, F\}$.

2. The cut with $X = \{S, C, D, F\}$ has capacity 15.

3. No, not true. Either find the max flow (which is 20), or notice that the capacity of a cut cannot be 19 (all the capacities are divisible by 3 except for 5), and use the Ford-Fulkerson theorem.

4. The capacity of the cut is 19, max $m(f) = 18$, min cut: $X = \{S, A, B, G, H\}$.

5. Yes: e must be in the minimum cut.

6. True (we can use augmenting paths of smaller values).

7. The s,t-cut with $X = V \setminus \{t\}$ is a minimum s,t-cut.

8. The min s,w-cut has capacity at least 100.

9. a) True.
   b) Not true, if there are two edge-disjoint minimum cuts.
   c) The edges which are in all the minimum cuts satisfy b). The algorithm for finding them is like in b).