Exercise-set 2.
Solutions

1. (a) yes, yes,
   (b) no, no,
   (c) no, yes,
   (d) no, yes.

2. 8 (the smallest possible and also realizable degree-sequence is 0,1,2,3,4,6).

3. There are $n-1$ odd degrees $\implies$ $n-1$ is even, n is odd $\implies$ one even degree in $\overline{G}$.

4. The number of edges between the vertices of degree ≤ 7 and the vertices of degree ≥ 16 is both ≤ 70 and ≥ 70 $\implies = 70$ $\implies$ $|E(G)| = 115$.

5. Count the number of edges between the k vertices of highest degree and the rest in two ways.

6. There are 11 of them.

7. There are 2 of them.

8. There are 1, 2 3 an 6 of them, respectively.

9. a) There are 4 such graphs,
   b) there are 4 such graphs,
   c) there are 2 such graphs.

10. a) Yes, yes, no (should have 15/2 edges).
    b) No (should have 11 vertices and 55/2 edges).

11. 4.

12. a) 8,
    b) 2.

13. a) no, b) yes.

14. a) and b) are isomorphic, and c) is not isomorphic to them.

15. a) and b) are isomorphic to it, c) is not.

16. No. The underlying graph is a cycle,...

17. a) For non-simple graphs no.
    b) For simple graphs yes: they cannot have (at least) 2 components.

18. The graph cannot have 3 components.

19. There are at most 2 components.

20. If G is not connected, then the edges between the components of it make $\overline{G}$ connected.

21. In every connected component of a graph there is an even number of odd-degree vertices.