Exercise-set 10.
Solutions

1. a) True.
   b) True.
   c) False.
   d) We get the same answers as for a), b), c).

2. a) max $m(f) = 27$,
   b) max $m(f) = 16$, min cut: $X = \{S, A, C', E\}$.

3. a) 3, 3,
   b) 3, 3,
   c) 4, 4.

4. a) 5,
   b) 7.

5. a) $\kappa(G) = 3,\ \lambda(G) = 3$,
   b) $\kappa(G) = n,\ \lambda(G) = n$.

6. $\kappa(G) = 12,\ \lambda(G) = 12$.

7. $k \leq \text{mindeg}(G)$.

8. Use Dirac's theorem.

9. a) NO (counterexample),
   b) TRUE (check cases of the definition).

10. The graph is K_{10} minus 5 edges. At least 4-vertex-connected.

11.

12. Use Menger's theorem. Of the 3 cycles obtained from the 3 paths between two vertices one must be even.

13.

14. Use Menger's theorem. (We add the edge either within a class or between the two classes.)

15. There are at least $n + k - 2$ paths of lengths 2 between any 2 vertices.

16. Use Menger's theorem. The 2 paths form a cycle.

17. For any two vertices we need at least 3 edges to cover all the paths between them.

18. a) Check cases of the definition.
    b) Add two new vertices, and connect them to the vertices of A an B.

19. Since $\kappa(G) \leq \lambda(G) \leq 3$ it is enough to check that
    -if $\kappa(G) = 1$ then G is not 2-edge-connected, and
    -if $\kappa(G) = 2$ then G is not 3-edge-connected.

20. a) Use Menger's theorem.
    b) No (counterexample).

21. The average degree is less than 4. 3-vertex connectivity is possible (example).

22. Use the flow algorithm with capacities 1 for each pair of vertices.