Exercise-set 1.
Solutions

1. $20!/8! \cdot 5! \cdot 7!$.
2. $2 \cdot (n!)^2$
3. $\binom{8}{4} \cdot \binom{6}{4} \cdot 4! = (8 \cdot 7 \cdot 6 \cdot 5) \cdot (6 \cdot 5 \cdot 4 \cdot 3)/4!$.
4. $\binom{n+m}{n}$.
5. $9 \cdot 10^3 - 7 \cdot 8^3$.
6. $\binom{2000}{4}$.
7. 5^{25}.
8. $\binom{15}{10}$.
9. $\binom{7}{3}$.
10. $\binom{24}{20}$.
11. $\binom{n-k}{k} \cdot k! \cdot 2^k$.
12. $\binom{15}{3} \cdot \binom{12}{4} \cdot 2^4$.
13. $\binom{2n}{2} \cdot \binom{2n-2}{2} \cdot \cdots \cdot \binom{2}{2}$.
14. $\binom{10}{3} \cdot \binom{7}{3} \cdot \binom{4}{2}\binom{2}{2}/2 \cdot 2$.
15. $\binom{30}{5} \cdot \binom{25}{5} \cdot \cdots \cdot \binom{10}{5}/6!$.
16. a) $\binom{\binom{20}{4}+4}{5}$,
    b) $\binom{\binom{20}{4}+4}{5} - \binom{\binom{19}{4}+4}{5}$.
17. a) At most 8, in 8! ways.
    b) At most 14, in 2^8 ways.
18. a) Equal.
    b) Equal.
19. a) $i\binom{n}{i} = n\binom{n-1}{i-1}$.
    b) We select n elements of a set of size $n+n$ (in 2 ways).
20. a) $\binom{12}{5}$.
    b) $\binom{12}{5} - \binom{7}{3}\binom{5}{2}$.
21. $\binom{n}{4}$.
22. 2^{100}.
23. $4 \cdot 10^6 - 4 \cdot 9^6$.