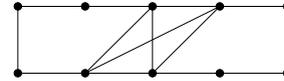
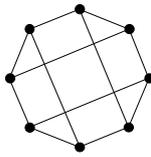
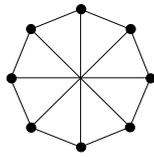
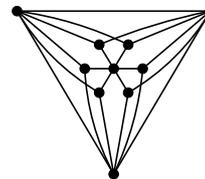
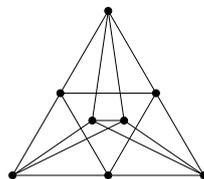


**Exercise-set 6.**

1. Determine whether the first two graphs below are bipartite or not:



2. At least how many edges must be deleted from the third graph above to get a bipartite graph?
3. 7 knights are put on a chessboard in such a way that each of them attacks at least two others. Show that there is such a knight among them which attacks three others.
4. Let the vertices of the graph  $G$  be the all the 0-1 sequences of length 5, and two sequences be adjacent if they differ in exactly one position. Is  $G$  a bipartite graph?
5. Is there a simple bipartite graph on at least 5 vertices whose complement is also a bipartite graph?
6. In a graph on 99 vertices two vertices have degree 3, and the degree of the other vertices is 4. Show that the graph contains an odd cycle.
7. Determine all the nonisomorphic simple graphs  $G$  on 8 vertices for which  $\chi(G) = 2$  but if we add any edge to  $G$  (between two nonadjacent vertices) then for the graph  $G'$  obtained this way  $\chi(G') = 3$  holds.
8. Determine all the nonisomorphic simple graphs  $G$  on  $n$  vertices for which  $\chi(G) = 3$  but if we delete any vertex from  $G$  (together with the edges adjacent to it) then for the graph  $G'$  obtained  $\chi(G') = 2$  holds.
- 
9. Determine the chromatic number of the graph of the regular octahedron. (The octahedron has 6 vertices and 8 triangular faces.)
10. Let the vertices of the graph  $G$  be the squares of the chessboard, and two vertices be adjacent if and only if the corresponding squares can be reached from each other by one move of a rook. Determine  $\chi(G)$ , the chromatic number of  $G$ . (A rook in chess can move either horizontally or vertically, and in one move it can go to any square along the selected line.)
11. Let the vertices of the graph  $G$  be the integers  $1, 2, \dots, 100$ , and two vertices,  $m$  and  $n$  be adjacent if and only if  $m + n$  is odd. Determine  $\chi(G)$ , the chromatic number of  $G$ .
12. Determine the chromatic number of the graphs below:



13. Let  $G$  be the graph obtained from a regular 11-sided polygon by adding all the shortest diagonals to it (i.e.  $G$  has 11 vertices and 22 edges). Determine  $\chi(G)$  and  $\omega(G)$ .
14. Determine the chromatic number of the complement of the cycle on  $n$  vertices.
15. Let the vertices of the graph  $G$  be the numbers  $1, 2, \dots, 2015$ , and two vertices be adjacent if and only if the difference of the corresponding numbers is at most 9. Determine  $\chi(G)$ , the chromatic number of  $G$ .
16. Let the vertex set of the graph  $G$  be  $V(G) = \{1, 2, \dots, 30\}$ . Let the vertices  $x, y \in V(G)$  be adjacent in  $G$  if the difference of the numbers  $x$  and  $y$  is at least 7. Determine  $\chi(G)$ , the chromatic number of  $G$ .
17. Let the vertex set of the graph  $G$  be  $V(G) = \{1, 2, \dots, 100\}$ . Let the vertices  $x, y \in V(G)$  be adjacent in  $G$  if  $x \neq y$  and  $100 \leq x \cdot y \leq 400$ . Determine the value of  $\chi(G)$ .

18. Let the vertices of the graph  $G$  be the numbers  $1, 2, \dots, 15$ , and two vertices be adjacent if and only if one of the corresponding numbers divides the other. Determine  $\chi(G)$ , the chromatic number of  $G$ .
  19. Let the vertices of the graph  $G$  be the numbers  $1, 2, \dots, 30$ , and two vertices be adjacent if and only if one of the corresponding numbers is relatively prime to the other. Determine  $\chi(G)$ , the chromatic number of  $G$ .
  20. In a simple graph  $G$  on 10 vertices the degree of each vertex is 8. Determine the chromatic number of  $G$ .
  21. (MT'19) We delete the edges of two cycles without common vertices (one of length 3 and one of length 4) from the complete graph on 10 vertices. Determine  $\chi(G)$ , the chromatic number of the graph  $G$  obtained this way.
  22. (MT+'19) From the complete graph on 12 vertices we delete 3 vertex-disjoint cycles of length 4. Determine the chromatic number of the graph obtained this way.
  23. (MT++'19) From a complete graph on 10 vertices we delete the edges of two such cycles on 3 vertices which have exactly one vertex in common. Determine the chromatic number of the graph obtained.
  24. a) Prove that  $|E(G)| \geq \binom{\chi(G)}{2}$  holds for every graph  $G$ .  
b) Prove that in a coloring a graph  $G$  with  $\chi(G)$  colors every color class contains a vertex  $v$  such that  $v$  has a neighbor in every other color class.
  25. Let  $G_1 = (V, E_1)$  and  $G_2 = (V, E_2)$  be two graphs on the same vertex set, and let  $G = (V, E_1 \cup E_2)$ . Prove that  $\chi(G) \leq \chi(G_1)\chi(G_2)$ .
- 
26. Let the vertex set of the graph  $G$  be  $V(G) = \{1, 2, \dots, 2015\}$ . Suppose that every vertex of  $G$  is adjacent to at most 10 smaller numbers. Prove that  $\chi(G) \leq 11$ .
  27. In the simple graph  $G$  apart from 100 exceptional vertices the degree of each vertex is at most 99. Prove that  $\chi(G) \leq 100$ .
  28. A simple graph  $G$  on 10 vertices contains one vertex of degree 5, one of degree 4, one of degree 3, and the rest of the vertices have degree 2. Show that  $G$  can be colored with 3 colors.
  29. Prove that every simple planar graph can be colored with 6 colors (without using the four color theorem).