Exercise-set 1.

1. We write '1' on eight out of twenty identical slips of paper, '2' on five and '3' on seven on them. In how many ways can we arrange these on a line?
2. How many orderings are there of the numbers $1, 2, \ldots, 2n$ in which the even and odd numbers alternate?
3. In how many ways can we reach the point (n, m) in the coordinate system starting from the origin if in each step we move either 1 to the right or 1 upward?
4. How many four-digit numbers are there in which at least one of the digits 5 and 6 occurs at least once?
5. How many strictly monotonically decreasing sequences of length four can be selected from the set $\{1, 2, ..., 2000\}$?
6. In how many ways can we send 25 different postcards to 5 of our friends during the summer? (Possibly some of them don't get any, even one of them can get them all.)
7. We roll 10 identical dices simultaneously. How many outcomes can we get?
8. In how many ways can we choose 3 scoops of icecream from 5 different flavors if in the bowl the order doesn't matter?
9. In how many ways can Santa Claus distribute 20 identical chocolates to 5 children? (There is no rule for the distribution of chocolates, even one child can get all of them. We consider two cases different if there is a child who got a different number of chocolates.)
10. In how many ways can we select four (mixed) couples out of a company of eight men and six women?
11. In how many ways can k married couples be seated in a row of n chairs such that the pairs sit next to each other?
12. In how many ways can we choose 10 people out of the members of 15 married couples in such a way that we choose exactly 3 couples?
13. In how many ways can $2n$ people of different height stand in 2 rows (of length n) behind each other in such a way that every person in the back row is taller than the one in front of her/him?
14. In how many ways can 10 people be divided into 2 groups of three and 2 groups of two?
15. In how many ways can a class of 30 be divided into 6 groups of size 5?
16. The local government of a small town has 20 members. They want to set up 5 committees of 4 members each to handle actual problems.
    a) In how many ways can the committees be set up?
    b) In how many ways can the committees be set up if they want to ensure that the mayor (who is one of the 20 members) should be on at least one committee?
    (There is no other restriction on the committees, so one person can serve on more than one committees, and two committees can consist of the same members as well.)

17. a) At most how many rooks can be placed on the chessboard in such a way that none of them attacks another one? In how many ways can we place the maximal number of rooks?
b) (∗) What about the same questions for bishops?

18. a) Does a set of size 99 have more even or odd subsets?
b) Does a set of size 100 have more even or odd subsets?

19. Show that for any positive integer n
a) $\sum_{i=1}^{n} i\binom{n}{i} = n \cdot 2^{n-1}$,
b) $\sum_{i=2}^{n} \binom{n}{i}^2 = \binom{2n}{n}$.

20. In how many ways can you read the word COMBINATORICS from the tables below if we can move only down and to the right?

a)

C	O	M	B	I	N	A	T
O	M	B	I	N	A	T	O
M	B	I	N	A	T	O	R
B	I	N	A	T	O	R	I
I	N	A	T	O	R	I	C
N	A	T	O	R	I	C	S

b)

C	O	M	B	I	N	A	T
O	M	B	I	N	A	T	O
M	B	I	N	A	T	O	R
B	I	N	A	$*$	O	R	I
I	N	A	T	O	R	I	C
N	A	T	O	R	I	C	S

21. (∗) There is no such point in a convex n-sided polygon where more than two diagonals of the polygon pass through. How many points are there in the polygon where the diagonals intersect?

22. (∗) At most how many subsets are there in a set of size 101 such that every two subsets have a common element?

23. (MT'18) How many sequences of length 99 can be made using the letters a, b, c, d, e which contain neither adjacent vowels nor adjacent consonants?

24. (MT++'18) How many seven-digit telephone numbers are there whose first digit is one of the numbers 1, 2, 3, 4, and the first digit is repeated at least once more in the number?

25. (MT'19) We want to choose a password for our e-mail account consisting of 8 characters, 3 of which are (not necessarily different) digits (from 0 to 9), and the remaining 5 characters are different letters (from the 26 letters of the English alphabet), such that exactly two of them are uppercase. How many passwords can we choose with these conditions?

26. (MT++'19) The neptun code of a student is a sequence consisting of 6 characters, each of which is either one of the 26 letters of the English alphabet or one of the digits $0, 1, \ldots, 9$. How many neptun codes are there which contain the letter A, but not the digit 1?