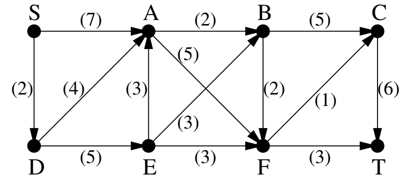
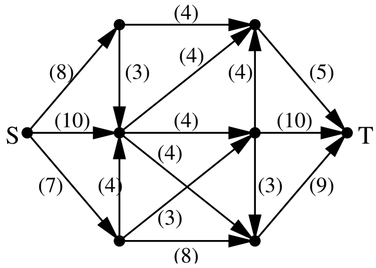
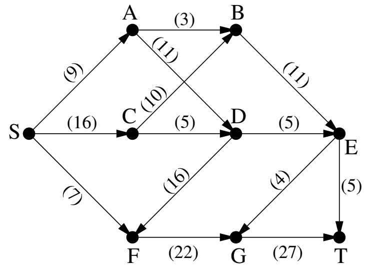
Combinatorics and Graph Theory 1.

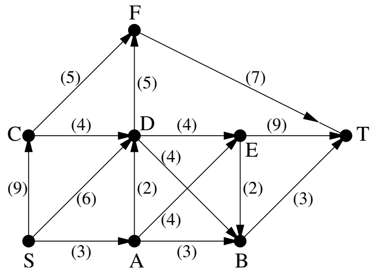
Exercise-set 9.

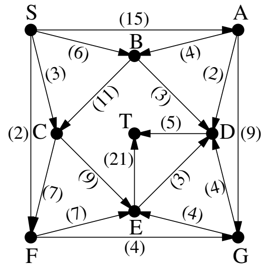
1. Determine the value of a maximum flow in the networks below, and prove that they are maximal.

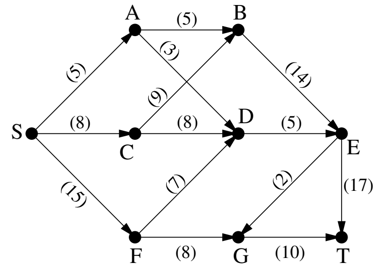


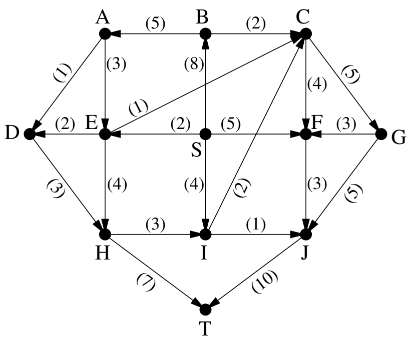


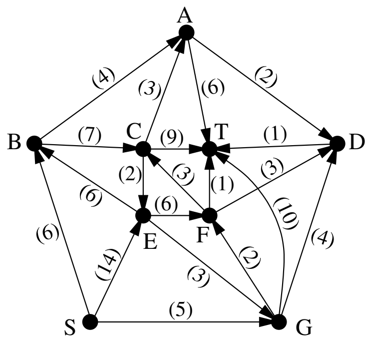




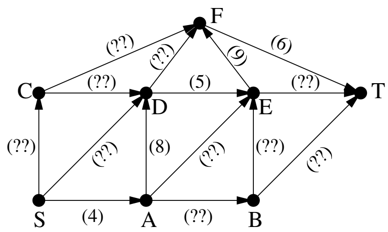




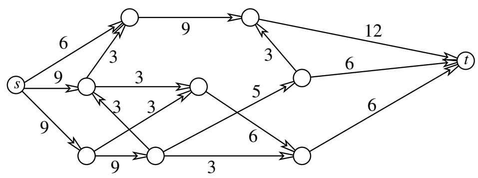




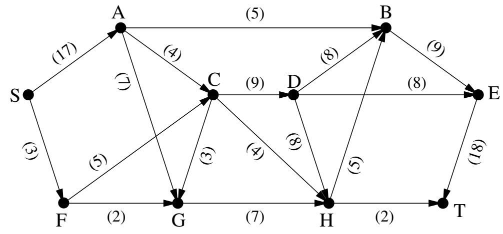
2. I just found a flow of value 15 in the network below when I spilled my coffee and the capacities of most of the edges became unreadable. Show that the flow I found was maximal.



3. Is it true that in the network (G, s, t, c) in the picture below the maximum flow value is exactly 19? (The numbers on the edges denote the appropriate capacities.)



4. Determine the capacity of the cut between S, A, G and the rest of the vertices in the network below and determine whether this cut is minimum or not (between S and T).



5. In a network the capacity of the edge e is 3, the capacities of all the other edges are 2, and we know that the value of the maximum flow f is an odd integer. Is it true then that $f(e) = 3$?

6. In a network with rational capacities the value of the maximum flow is m. Is it true then that for each value $0 \leq x \leq m$ there is a flow of value x in this network?

7. Let a directed graph G, the vertex $s \in V(G)$ and the capacity function $c : E(G) \to \mathbf{R}^+$ be given. For all $v \in V(G),\ v \neq s$ let $m(v)$ denote the value of the maximum flow from s to v. Suppose that for some vertex $t \in V(G),\ m(t) = 100$ holds, but for every vertex $v \in V(G),\ v \neq s, t,\ m(v) > 100$. Show that in this case the total capacity of the edges arriving into t is 100.

8. Let a directed graph G and the capacity function $c : E(G) \to \mathbf{R}^+$ be given. Suppose that for the vertices s, t and $w \in V(G)$ there is a flow of value 100 from s to t and also from t to w. Prove that there exists a flow of value 100 from s to w as well.

9. Decide whether the following statements are tue or not.
a) Every network contains an edge e such that if we decrease its capacity by ε (where $0 < \varepsilon \leq c(e)$) then the value of the maximum flow decreases by ε.
b) Every network contains an edge e such that if we increase its capacity by a positive ε then the value of the maximum flow increases by ε.
c) If one of the the statements above doesn't hold always, then what is the condition for a network to satisfy it? Give an algorithm to decide whether a network contains such an edge or not.