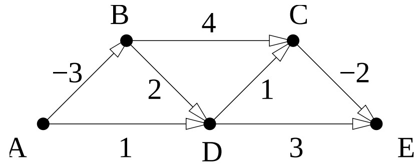
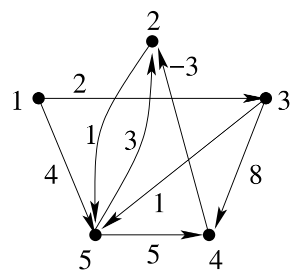
Combinatorics and Graph Theory 1.

Exercise-set 13.

1. Determine the shortest paths from vertex A to the other vertices in the graph below using Ford's algorithm.

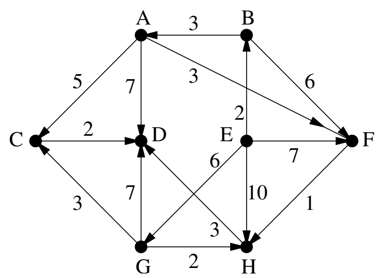


2. We use Floyd's algorithm on the graph below to determine the lengths of the shortest paths from x to y for all the possible pairs of vertices (x, y). During the algorithm (at the end of the 4th round) the known (upper bounds on the) distances are contained in the matrix A_4 below.
a) Determine the next matrix for the algorithm.
b) What are the distances between the pairs of vertices?

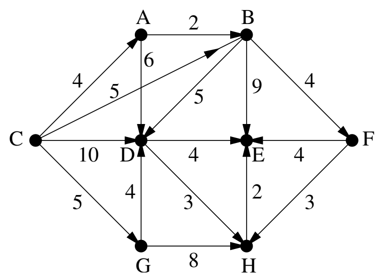


$$A_4 = \begin{pmatrix} 0 & 7 & 2 & 10 & 3 \\ \infty & 0 & \infty & \infty & 1 \\ \infty & 5 & 0 & 8 & 1 \\ \infty & -3 & \infty & 0 & -2 \\ \infty & 2 & \infty & 5 & 0 \end{pmatrix}$$

3. a) Run the DFS algorithm on the graph below starting from the vertices A, and E. Determine the depth numbers, the backtrack numbers and the DFS forest obtained.
b) Determine whether the graph is acyclic or not, and if yes, then determine a topological ordering of the vertices.
c) Compute the lengths of the shortest and longest paths form vertex E to the other vertices.



4. a) Determine whether the graph below is acyclic or not, and if yes, then determine a topological ordering of the vertices.
b) Compute the lengths of the shortest and longest paths form vertex A to the other vertices.



5. Let the vertices of the connected, undirected graph G be denoted by x, y, z, u, v, w. After running the DFS algorithm on G, we get the following depth and backtrack numbers for the vertices: $x : 1, 6;\ y : 2, 4;\ z : 6, 5;\ u : 3, 3;\ v : 4, 1;\ w : 5, 2$.
a) Determine the edges of the spanning tree belonging to search.
b) Can we reconstruct G from the given depth and completion numbers?
c) At most how many edges can G contain?

6. There is a black-and-white image consisting of $n \times n$ pixels. We would like to draw a line from the top left corner to the lower right corner of the picture going right/down in such a way that the sum of the number of black pixels to the right (top) of the line and the number of white pixels to the left left (down) of the line is as small as possible. (The line goes between the pixels everywhere.) Give an algorithm solving the problem in at most $c \cdot n^2$ steps.

7. Let G be a connected, undirected graph. Is it true that
   a) for every edge f of G there is a DFS started from some vertex, in which f is a tree-edge?
   b) for every edge f of G there is a BFS started from some vertex, in which f is a tree-edge?
   c) for every edge f of G there is a DFS started from every vertex, in which f is a tree-edge?
   d) for every edge f of G there is a BFS started from every vertex, in which f is a tree-edge?
   e) for every spanning tree F of G there is a DFS in which F is the DFS spanning tree obtained?
   f) for every spanning tree F of G there is a DFS in which F is the BFS spanning tree obtained?

8. Show that the edge set of every loop-free directed graph can be decomposed into two sets which both determine an acyclic directed graph (on the original vertices).

9. Let G be a connected, undirected graph with 10 vertices and 10 edges. Is it true that every spanning tree of G is a DFS spanning tree of G?

10. Give four, pairwise non-isomorphic simple undirected graphs on 6 vertices for which each of the DFS spanning trees are paths.