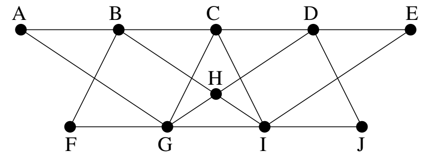
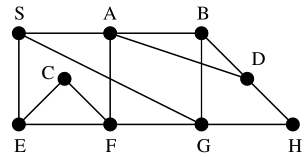
Exercise-set 11.

1. Can the vertices of the graph below be reached in the following order using the BFS algorithm?
a) H, B, D, G, I, C, A, F, J, E
b) F, B, A, G, C, H, I, D, E, J
c) J, D, I, C, E, G, H, A, F, B
d) A, B, G, C, H, F, I, D, E, J



2. The BFS algorithm visited the vertices of the graph below in the following order: $S, \square, \square, \square, H, \square, F, C, \square$.
a) Complete the sequence with the missing vertices (which are denoted by $\square$), and determine the corresponding BFS tree.
b) Can the edge $\{D, H\}$ be contained in an arbitrary BFS spanning tree started from S?



3. We want to decide for a given graph G and vertex s whether there is a cycle in G containing s and if yes then we want to find a shortest such cycle. Modify the BFS algorithm so that it can solve this problem as well.

4. In the connected graph G the degree of each vertex is 3. We start a BFS algorithm from vertex s which reaches vertex v in the 13th place (we consider s to be the vertex first reached). Is it possible that the distance of v from s is
a) 2,
b) 3,
c) 8?

5. We call the spanning tree F of a connected graph G *suitable* for a vertex v of G, if there is a BFS started from v which is exactly F. At most how many edges can a connected graph G on 100 vertices have if it has a spanning tree which is suitable for every vertex of G?

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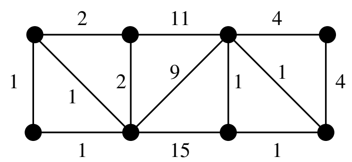
6. Determine a minimum weight spanning tree in the weighted graph below. How many such trees are there?



7. Let G be the complete graph on the vertex set $V(G) = \{1, 2, \ldots, 100\}$. For every $1 \leq i, j \leq 100$, $i \neq j$ let the weight of the edge $\{i, j,\}$ be the larger of the values of i and j. What is the weight of a minimum weight spanning tree in G? Determine such a tree. How many minimum weight spanning trees are there?

8. Let G be the complete graph on the vertex set $V(G) = \{1, 2, \ldots, 100\}$. For every $1 \leq i, j \leq 100$, $i \neq j$ let the weight of the edge $\{i, j,\}$ be 1, if $i, j \leq 50$, 2, if $i, j \geq 51$, and 3 for all the other edges. What is the weight of a minimum weight spanning tree in G? Determine such a tree.

9. Let G be a connected graph and $w : E(G) \to \mathbf{R}$ be a weight function on the edges of G. Suppose that one of the endpoints of the edge e of G is v and for all the edges f which are incident to v the inequality $w(e) \leq w(f)$ holds. Show that G has a minimum weight spanning tree which contains e.

10. Let G be a connected graph and $w : E(G) \to \mathbf{R}$ be a weight function on the edges of G. Furthermore, let C be a cycle in G and e an edge of C. Suppose that $w(e) \geq w(f)$ holds for all the edges f of the cycle C. Show that G has a minimum weight spanning tree which doesn't contain e.