Exercise-set 1.

1. How many orderings of the letters of MATHEMATICS are there?
2. How many orderings are there of the numbers $1, 2, \ldots, 2n$ in which the even and odd numbers alternate?
3. In how many ways can we select four (mixed) couples out of a company of eight men and six women?
4. In how many ways can 10 people be divided into 2 groups of three and 2 groups of two?
5. In how many ways can we reach the point (n, m) in the coordinate system starting from the origin if in each step we move either 1 to the right or 1 upward?
6. How many four-digit numbers are there in which at least one of the digits 5 and 6 occurs at least once?
7. How many strictly monotonically decreasing sequences of length four can be selected from the set $\{1, 2, ..., 2000\}$?
8. In how many ways can we send 25 different postcards to 5 of our friends during the summer? (Possibly some of them don't get any, even one of them can get them all.)
9. We roll 10 identical dices. How many outcomes can we get?
10. In how many ways can we choose 3 scoops of icecream from 5 different flavors if in the bowl the order doesn't matter?
11. In how many ways can k married couples be seated in a row of n chairs such that the pairs sit next to each other?
12. In how many ways can we choose 10 people out of the members of 15 married couples in such a way that we choose exactly 3 couples?
13. In how many ways can Santa Claus distribute 20 identical chocolates to 5 children? (There is no rule for the distribution of chocolates, even one child can get all of them. We consider two cases different if there is a child who got a different number of chocolates.)
14. In how many ways can $2n$ people of different height stand in 2 rows (of length n) behind each other in such a way that every person in the back row is taller than the one in front of her/him?
15. In how many ways can a class of 30 be divided into 6 groups of size 5?
16. a) At most how many rooks can be placed on the chessboard in such a way that none of them attacks another one? In how many ways can we place the maximal number of rooks?
    b) ($*$) What about the same questions for bishops?
17. a) Does a set of size 99 have more even or odd subsets?
    b) Does a set of size 100 have more even or odd subsets?
18. Show that for any positive integer n
    a) $\sum_{i=1}^{n} i\binom{n}{i} = n \cdot 2^{n-1}$,
    b) $\sum_{i=2}^{n} \binom{n}{i}^2 = \binom{2n}{n}$.
19. In how many ways can you read the word COMBINATORICS from the tables below if we can move only down and to the right?

a)

C	O	M	B	I	N	A	T
O	M	B	I	N	A	T	O
M	B	I	N	A	T	O	R
B	I	N	A	T	O	R	I
I	N	A	T	O	R	I	C
N	A	T	O	R	I	C	S

b)

C	O	M	B	I	N	A	T
O	M	B	I	N	A	T	O
M	B	I	N	A	T	O	R
B	I	N	A	$*$	O	R	I
I	N	A	T	O	R	I	C
N	A	T	O	R	I	C	S

20. ($*$) There is no such point in a convex n-sided polygon where more than two diagonals of the polygon pass through. How many points are there in the polygon where the diagonals intersect?
21. Show that in a class of 37 there are always 4 people who were born in the same month.

22. Prove that we can select some integers from a set of 2018 arbitrary integers such that their sum is a multiple of 2018.

23. (∗) At most how many subsets are there in a set of size 101 such that every two subsets have a common element?

24. How many two-digit integers are there which are not divisible by either 2 or 3 or 5?

25. n men put their hats in the cloakroom. In how many ways can they get them back at the end of the show if nobody gets his own hat back?

26. From a class of 30 some people (at least one) have to go to the CGT1 class, some people (at least one) to the students' office and some people (at least one) to the dining hall, all at the same time. In how many ways can they do it?

27. (∗) 10 robbers collect the hoard in a case which can be closed with lots of locks. They want to lock the case and distribute the keys in such a way that any 4 robbers could open the case, but no 3 robbers could do the same (more than one of them can get a key to a lock). At least how many locks do they need to do this?