Value-transformation for Monotone Prediction by Approximating Fuzzy Membership Functions

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Monotone Prediction

A labeling $l : \mathcal{X} \to \mathbb{N}$ is called monotone if for all pair of objects $\mathbf{x}, \mathbf{y} \in \mathcal{X} \subseteq \mathcal{X}_1 \times \cdots \times \mathcal{X}_k$ holds that *if* $\mathbf{x} \preceq \mathbf{y}$ *then* $l(\mathbf{x}) \leq l(\mathbf{y})$, where

$$\mathbf{x} \preceq \mathbf{y} \ \Leftrightarrow\ (\forall j \in \{1, \ldots, k\})\ x_j \leq_j y_j \qquad (1)$$

Recent Approaches

Monotone Prediction

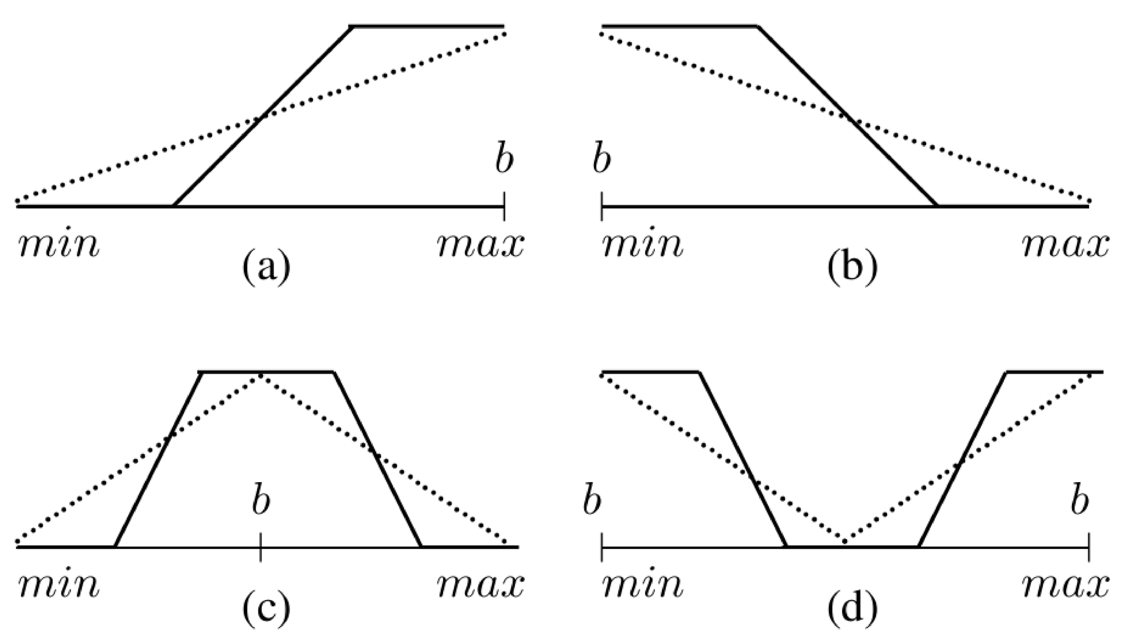
- **numerical attributes** only
- **linear ordering** $\leq_j$ on attribute domains, i.e. when the lower or higher values are the better
- Most of them **assume monotone training sets** without monotone noise

Data pre-processing

1. multiply attribute values with -1 if their negatively **correlate** with class values
2. if there is still some monotone noise, **relabel** non-monotone pairs of objects

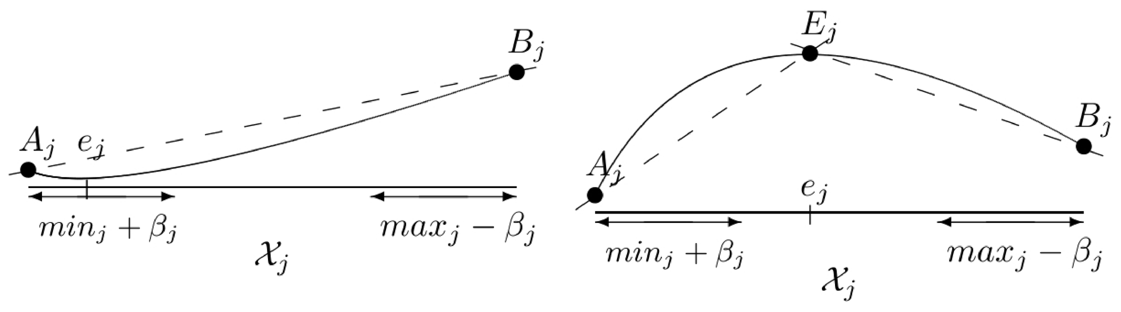
Our Approach

Replace $\leq_j$ in equation (1) by fuzzy membership functions $f_j : \mathcal{X}_j \to \mathbb{R}$ approximated from data.



Algorithm

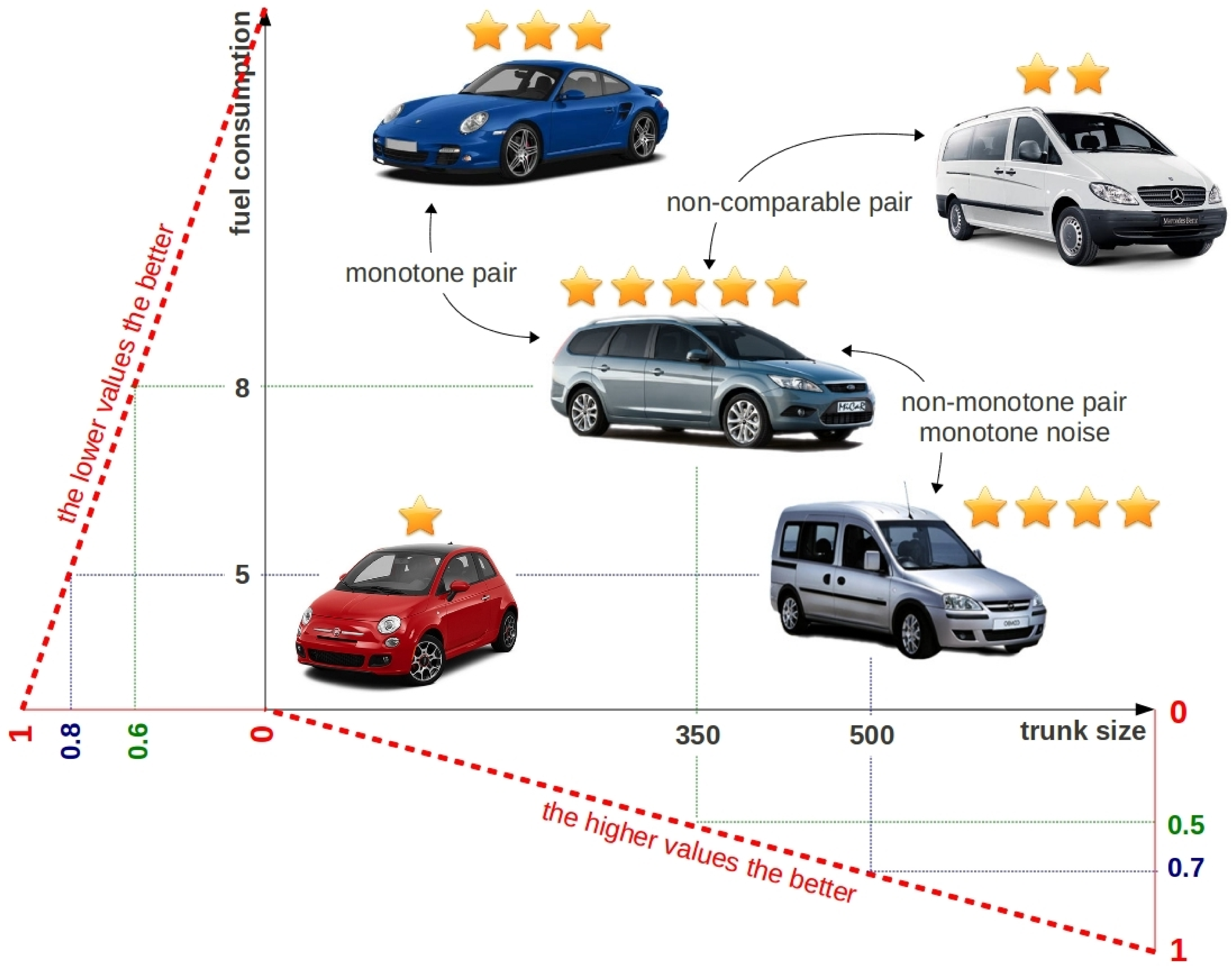
1. compute a parabola $p_j(X_j) = \alpha X_j^2 + \beta X_j + \gamma$ from the projection $\pi_j \subset \mathcal{X}_j \times \mathbb{N}$ of the values of the jth attribute to class labels
2. compute $A = p_j(min_j), B = p_j(max_j)$ and $E = p_j(e_j)$, where e_j is the extreme of p_j
3. if e_j lies on the border of the domain or outside the domain then $\hat{f}_j$ is a line defined by the points A_j and B_j. If e_j lies in the middle of the domain then $\hat{f}_j$ consists of two lines $\hat{f}_{jAE}, \hat{f}_{jEB}$ defined by the points A_j, E_j and E_j, B_j



In case of nominal attributes, we compute $\hat{f}_j$ as

$$\hat{f}_j(x) = \frac{\sum_{\{\mathbf{x}^i \in \mathcal{X} | x^i_j = x\}} l(\mathbf{x}^i)}{|\{\mathbf{x}^i \in \mathcal{X} | x^i_j = x\}|}$$

Example



Inverse mappings

The conditions in bodies of **Monotone Decision Tree** rules can be one of the following form:

$$X_j = v, \quad X_j \geq v, \quad X_j \leq v$$

where $v \in \mathbb{R}$ is a transformed value.

To interpret these rules we use the inverse mappings $\hat{f}_j^{-1}$ depending on the type of $\hat{f}_j$.
For example, if $\hat{f}_j$ is of type (c), then the above conditions will be interpreted as

$$X_j = \hat{f}^{-1}_{jAE}(v) \wedge X_j = \hat{f}^{-1}_{jEB}(v)$$

$$X_j \geq \hat{f}^{-1}_{jAE}(v) \wedge X_j \leq \hat{f}^{-1}_{jEB}(v)$$

$$X_j \leq \hat{f}^{-1}_{jAE}(v) \wedge X_j \geq \hat{f}^{-1}_{jEB}(v)$$

Experiment (2)

Measuring the **influence of pre-processing** to Monotone Classification

Baselines: *orig*, *corr*

Monotone Classification **algorithm used**:
– W. Duivesteijn and A. Feelders: *Nearest Neighbour Classification with Monotonicity Constraints.* Proceedings of ECML/PKDD '08

10-fold cross-validation to measure the average accuracy of classification on 10 folds

Results:
– Algorithm computed only for 27 datasets
– in 17 cases *fuzz* outperformed *orig*
– in 14 cases *fuzz* outperformed *corr*
– in 2 cases, the improvement was significant

Experiment (1)

Baselines – use only the numerical attributes

- *orig*: no pre-processing
- *corr*: correlation-based pre-processing

Monotonicity degree δ of the transformed (pre-processed) datasets $\mathcal{X}'$ measured

$$\delta = \frac{\#\text{ of monotone pairs in } \mathcal{X}'}{\#\text{ of comparable pairs in } \mathcal{X}'}$$

Tested on 40 **datasets** from the UCI machine learning repository

Results

Name	#Obj	#Num/#Nom	δ_{orig}	δ_{corr}	δ_{fuzzy}
auto-mpg	398	7 / 2	0.204	0.977	**0.979**
breast-c.	699	10 / 1	1	1	1
communit.	1994	123 / 5	NaN	NaN	**1**
concrete	1030	9 / 0	0.953	**0.986**	0.984
forestf.	517	11 / 2	0.733	0.719	**0.775**
machine	209	7 / 3	0.918	0.954	**0.958**
servo	167	3 / 2	0.333	0.704	**0.85**
slump	103	8 / 3	NaN	1	1
wdbc	569	31 / 1	1	1	1
wine	178	13 / 1	0.333	1	1
wineq.-r	1599	12 / 0	0.82	0.965	**0.966**
wineq.-w	4898	12 / 0	0.786	**0.919**	0.899
wpbc	198	33 / 2	0.962	1	1
abalone	4177	8 / 1	0.829	0.829	**0.854**
adult	32561	7 / 8	0.966	0.965	**0.988**
agaricus	8124	1 / 22	–	–	**1**
austral.	690	15 / 0	0.977	0.991	0.991
bands	540	21 / 19	1	0.957	1
car	1728	7 / 0	0.902	1	1
cmc	1473	8 / 2	0.8	0.818	**0.845**
crx	690	7 / 9	0.923	0.963	**0.991**
diagnosis	120	2 / 6	0.677	0.813	**1**
haberman	306	4 / 0	**0.893**	0.878	0.874
heart	270	14 / 0	0.988	**0.996**	0.995
hepatitis	155	7 / 13	0.98	0.993	**0.996**
horse-c.	300	17 / 11	1	1	1
house-v.	435	1 / 16	–	–	**0.986**
ionosph.	351	35 / 0	1	1	1
magic04	19020	11 / 0	0.446	0.979	**0.985**
mammogr.	961	4 / 2	0.944	0.944	**0.969**
nursery	12960	2 / 7	0.694	0.694	**1**
parkins.	195	23 / 1	NaN	0.995	**0.997**
pima	768	9 / 0	0.978	**0.978**	0.977
poker	25010	11 / 0	0.693	**0.735**	0.734
post-oper.	90	9 / 0	0.772	0.825	**0.857**
shuttle-l.	15	7 / 0	0.833	0.833	**1**
spambase	4601	58 / 0	0.977	**0.998**	0.987
tae	151	2 / 4	0.644	0.677	**0.882**
tic-tac-t.	958	1 / 9	–	–	**0.961**
transfu.	748	5 / 0	0.807	**0.94**	0.936

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