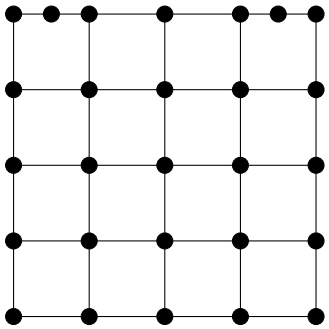
Introduction to Computer Science 2. **Midterm Test** 5/8/2020

1. In a simple graph on 20 vertices there are no isolated vertices, and there are exactly 3 vertices of degree 1. Show that the graph has at least 19 edges.

2. In a simple, non-connected graph the degree of each vertex is at least 2. Show that the complement of the graph is not planar.$K_{3,3}$

3. Decide whether the graph below contains a Hamilton cycle or not.



4. The bipartite graph G on 100 vertices contains a perfect matching. Determine the size of a minimum covering set of vertices in the graph H which we obtain from G by adding an arbitrary new edge to it.

5. From a graph on $2n$ vertices we delete the edges of a Hamilton cycle. Show that the edge-chromatic number of the graph cannot decrease by more than 2.

6. $(*)$ In a simple cycle-free graph on 12 vertices exactly 2 kinds of degrees occur, and both of them at least five times. How many edges can the graph have?

Total work time: 90 min + 30 minutes for uploading. Late turn-ins are not accepted.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.