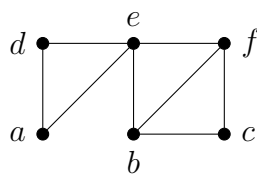
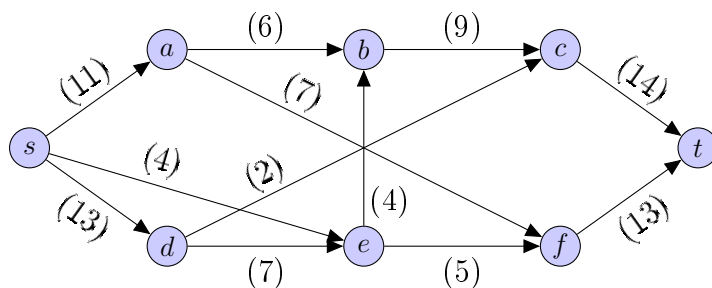


- Let  $G$  be a simple, connected planar graph. Show that if we delete the edges of two edge-disjoint spanning trees from  $G$  then the remaining graph cannot be connected.
- We add two non-adjacent edges to the complete bipartite graph  $K_{3,3}$  in such a way that the resulting graph  $G$  is simple. Determine  $\chi(G)$ , the chromatic number of  $G$ .
- In a tree  $T$  on 20 vertices 11 vertices have degree 1, and the degree of the remaining 9 vertices are the same as well. Determine the  $\chi_e(T)$ , edge-chromatic number of the tree.
- Decide whether the following graph is an interval graph or not.



- (\*) Give a graph whose chromatic number decreases by ten if we delete ten appropriate edges from it. Determine the minimum number of vertices of graphs with this property.
- Determine a maximum flow and a minimum  $s, t$ -cut in the network below.



Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.