Introduction to Computer Science **Test for the Signature** 5/23/2018

1. The neptun code of a student is a sequence consisting of 6 characters, each of which is either one of the 26 letters of the English alphabet or one of the digits $0, 1, \ldots, 9$. How many neptun codes are there which contain 3 letters and 3 digits and all the letters in it are different?

2. $(*)$ Determine all the trees on 25 vertices for which there exists an integer $m \geq 2$, such that the degree of each vertex gives the same remainder when divided by m.

3. Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 10\}$, and let $x, y \in V(G)$ be adjacent if and only if $|x - y| = 2$ or $|x - y| = 3$.
   a) Does G contain a Hamilton path?
   b) Does G contain a Hamilton cycle?

4. At least how many edges have to be deleted from the complete bipartite graph $K_{3,4}$ in such a way that the resulting graph is planar? (In other words, what is the number k, such that we can delete k appropriate edges from $K_{3,4}$ in such a way that the resulting graph is planar, but if we delete $k-1$ edges in any way then the resulting graph cannot be planar.)

5. The graph G on 200 vertices is constucted from two (vertex-disjoint) cycles on 100 vertices each in such a way that we connect each vertex of one cycle with every vertex of the other cycle. Determine $\chi_e(G)$, the edge-chromatic number of the graph G.

6. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_8\}$ and $B = \{b_1, b_2, \ldots, b_8\}$. For each $1 \leq i, j \leq 8$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine $\tau(G)$, the minimum number of covering vertices and $\rho(G)$, the minimum number of covering edges, and give a minimum covering set of vertices and a minimum covering set of edges in G.

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used.