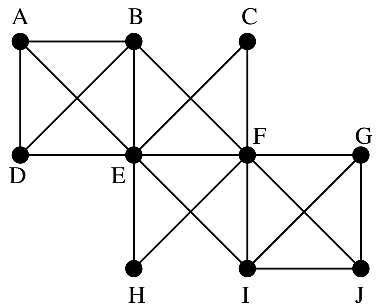
Introduction to Computer Science 2. 4/28/2016

Second Midterm Test

1. Let the vertex set of the graph G on 9 vertices be the vertices of the unit cube together with the center of it, i.e. $V(G) = \{(x, y, z) : x, y, z \text{ are 0 or 1}\} \cup \{(1/2, 1/2, 1/2)\}$. Let two vertices of G be adjacent if they differ either in the first or the second coordinate, or both. (E.g. $(0, 0, 1)$ is adjacent to $(0, 1, 1)$ and $(1, 1, 0)$ but not to $(0, 0, 0)$.) Determine $\chi(G)$, the chromatic number of G.

2. Delete 4 edges from the complete graph on 8 vertices, in such a way that all of them are incident to a given vertex. Determine whether the graph obtained is an interval graph or not.

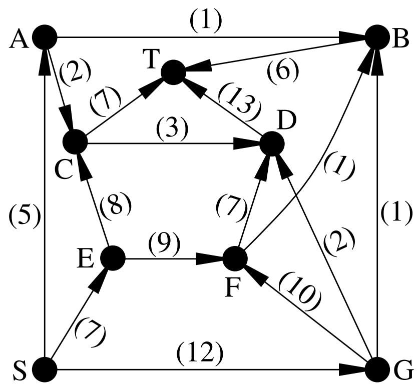
3. Use Tutte's theorem to prove that the graph below doesn't contain a perfect matching. (Tutte's theorem gives a necessary and sufficient condition for an arbitrary graph to contain a perfect matching.)



4. The vertex v of the simple graph G has degree 2, but all the other vertices of G have degree 3. Determine $\chi_e(G)$, the edge-chromatic number of G.

5. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_{101}\}$ and $B = \{b_1, b_2, \ldots, b_{101}\}$. For each $1 \le i \le 101$ and $1 \le j \le 101$ let a_i and b_j be adjacent if $i \cdot j$ is even. Determine $\nu(G)$, the maximum number of independent edges, $\rho(G)$, the minimum number of covering edges, and give a maximum matching and a minimum covering set of edges in G.

6. Determine a maximum flow in the network below (from S to T).



Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.