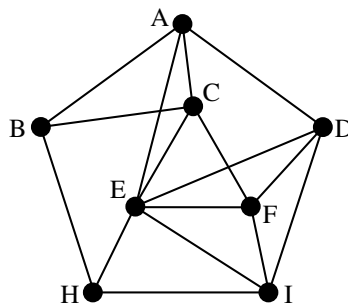


Repeated First Midterm Test, 5/9/2016

1. In an ice cream parlor 26 kinds of ice cream are sold. A customer orders 4 bowls of ice cream, all for himself. He wants 3 scoops in each of the 4 bowls, and within a bowl all different kinds. He doesn't mind getting the same flavor more than once in different bowls, but he doesn't want two bowls with 3 identical scoops. In how many ways can the customer be served? (Within a bowl the order of the scoops doesn't matter, neither does the order of the bowls, since all of them are ordered by the same customer.)
2. At least how many edges must a simple graph on 10 vertices have if it has 3 vertices of degree 9? (I.e., for which integer k does it hold that there is a graph with k edges having the above property, but for no graph with less than k edges has it?)
3. Does $K_{10,10}$, the complete bipartite graph on $10 + 10$ vertices have a spanning tree in which the degree of each vertex is odd?
4. Decide whether the following graph is planar or not. If yes, then draw it with straight edges without crossing, if not, then prove it.



5. a) Does the graph above have an Euler circuit? If yes, then give one, if no, then prove it.
b) Does the graph above have an Euler trail? If yes, then give one, if no, then prove it.
6. In a simple graph G on 50 vertices the degree of each vertex is at most 24. Show that we can assign the numbers from 1 to 50 to the vertices of G in such a way that the difference of the numbers assigned to neighboring vertices is at least 2.

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points.

Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.