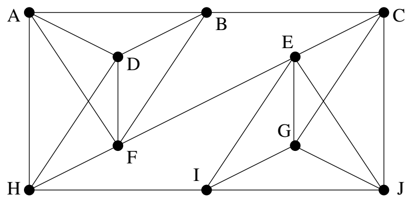
List of Exercises

1. At most how many pairwise edge-disjoint and vertex-disjoint paths are there between the following points in the graph below:
a) B and G, b) A and I, c) B and H.

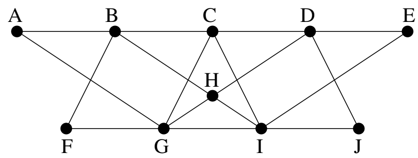


2. (MT'12) The graph G on 15 vertices is constructed from three cycles, on 4, 5 and 6 vertices each, in such a way that each vertex of the 5-vertex cycle was connected (with one edge) to all the other vertices of the other two cycles. Let s be a vertex of the 4-vertex cycle, and t be a vertex of the 6-vertex cycle.
a) At most how many pairwise vertex-disjoint paths are there in G between s and t?
b) At most how many pairwise edge-disjoint paths are there in G between s and t?

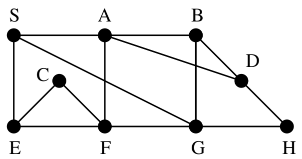
3. Determine the vertex- and edge connectivity numbers ($\kappa(G)$ and $\lambda(G)$) of the following graphs:
a) a path on 100 vertices,
b) a cycle on 100 vertices,
c) the graph consisting of the vertices and edges of a cube,
d) the complete bipartite graph $K_{m,n}$, where $m \geq n$.

4. The vertices of an 18-vertex graph G can be divided into 3 classes of six vertices each, in such a way that 2 vertices are adjacent if and only if they are in different classes. Determine the largest integer k for which G is k-vertex-connected ($\kappa(G)$), and the largest integer l for which G is l-edge-connected ($\lambda(G)$).

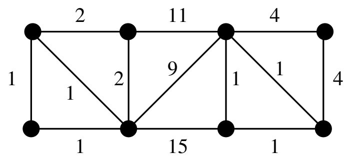
5. Can the vertices of the graph below be reached in the following order using the BFS algorithm? If yes, then determine the corresponding BFS-tree and the distances of the vertices from the starting vertex.
a) J, D, I, C, E, G, H, A, F, B b) A, B, G, C, H, F, I, D, E, J



6. (MT'15) The BFS algorithm visited the vertices of the graph below in the following order:
$S, \square, \square, \square, H, \square, F, C, \square$.
a) Complete the sequence with the missing vertices (which are denoted by $\square$), and determine the corresponding BFS tree.
b) Can the edge $\{D, H\}$ be contained in an arbitrary BFS spanning tree started from S?



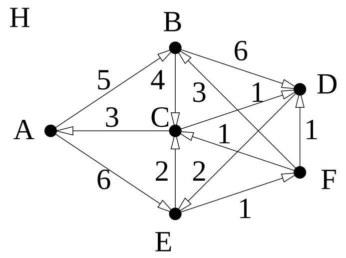
7. Determine a minimum weight spanning tree in the weighted graph below. How many such trees are there?



8. Let G be the complete graph on the vertex set $V(G) = \{1, 2, \ldots, 100\}$. For every $1 \leq i, j \leq 100$, $i \neq j$ let the weight of the edge $\{i, j, \}$ be the larger of the values of i and j. What is the weight of a minimum weight spanning tree in G? Determine such a tree.

9. Let G be the complete graph on the vertex set $V(G) = \{1, 2, \ldots, 100\}$. For every $1 \le i, j \le 100$, $i \ne j$ let the weight of the edge $\{i, j,\}$ be 1, if $i, j \le 50$, 2, if $i, j \ge 51$, and 3 for all the other edges. What is the weight of a minimum weight spanning tree in G? Determine such a tree.

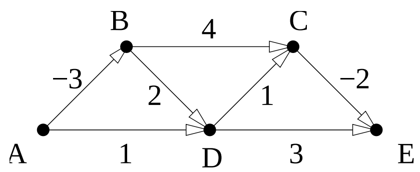
10. a) Determine the shortest paths from vertex A to the other vertices in the first graph below using Dijkstra's algorithm.
b) We decrease the weight of one edge by 1. For which edges will the distances from vertex A remain the same?
c) We add the edge (B, E) to the graph. For which weights $l(B, E) \ge 0$ will the lengths of the shortest paths change?



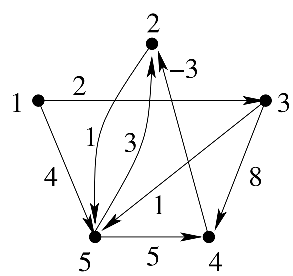
11. Determine all the directed graphs with the minimum number of edges for which the table below gives the changes of the array containing the values $d(v)$(= the actual distance of v from v_1) in Dijkstra's algorithm. Determine the states of the array $p(v)$(= vertex preceding v) as well.

v_1	v_2	v_3	v_4	v_5	v_6
0	2	6	∞	∞	7
0	2	5	9	∞	6
0	2	5	6	9	6
0	2	5	6	8	6
0	2	5	6	7	6

12. Determine the shortest paths from vertex A to the other vertices in the graph below using Ford's algorithm.

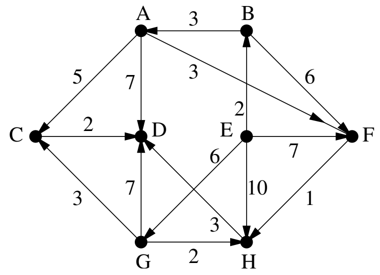


13. We use Floyd's algorithm on the graph below to determine the lengths of the shortest paths from x to y for all the possible pairs of vertices (x, y). During the algorithm (at the end of the 4th round) the known (upper bounds on the) distances are contained in the matrix A_4 below.
a) Determine the next matrix for the algorithm.
b) What are the distances between the pairs of vertices?

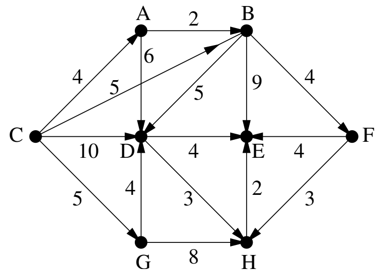


$$A_4 = \begin{pmatrix} 0 & 7 & 2 & 10 & 3 \\ \infty & 0 & \infty & \infty & 1 \\ \infty & 5 & 0 & 8 & 1 \\ \infty & -3 & \infty & 0 & -2 \\ \infty & 2 & \infty & 5 & 0 \end{pmatrix}$$

14. a) Run the DFS algorithm on the graph below starting from vertex C. Determine the depth numbers, the completion numbers and the DFS forest obtained.
b) Determine whether the graph is acyclic or not, and if yes, then determine a topological ordering of the vertices.
c) Compute the lengths of the shortest and longest paths form vertex E to the other vertices.



15. a) Determine whether the graph below is acyclic or not, and if yes, then determine a topological ordering of the vertices.
b) Compute the lengths of the shortest and longest paths form vertex A to the other vertices.



16. Let the vertices of the connected, undirected graph G be denoted by x, y, z, u, v, w. After running the DFS algorithm on G, we get the following depth and completion numbers for the vertices: $x: 1, 6;\ y: 2, 4;\ z: 6, 5;\ u: 3, 3;\ v: 4, 1;\ w: 5, 2$.
a) Determine the edges of the spanning tree belonging to search.
b) Can we reconstruct G from the given depth and completion numbers?
c) At most how many edges can G contain?