List of Questions

1. Basic notions of graph theory: graph, simple graph, degree, isomorphism, complement, subgraph, walk, trail, circuit, path, cycle, connectedness, components. Breadth First Search, number of steps it makes.
2. Trees: basic properties**, spanning trees, their existence**. Minimum weight spanning trees, Kruskal's algorithm**.
3. Euler trail and circuit, necessary and sufficient conditions for their existence**. Hamilton path and cycle, necessary conditions**, sufficient conditions: Dirac's** and Ore's** theorem.
4. Vertex coloring: the notion of $\chi(G)$ and its relationship to $\omega(G)$** and $\Delta(G)$**. Zykov's construction**. Greedy coloring**. Interval graphs, their coloring**.
5. Bipartite graphs, relationship with odd cycles**. Covering and independent vertices and edges, relations between them. Gallai's theorems**.
6. Matchings in bipartite graphs. Augmenting paths. Theorems of König**, Hall** and Frobenius**. Edge-chromatic number, its relation to $\Delta(G)$**. Vizing's theorem, Shannon's theorem. König's theorem** (about edge-chromatic number of bipartite graphs).
7. Network, flow, value of a flow, s-t cut, capacity of a cut, augmenting paths. Ford-Fulkerson theorem**, Edmonds-Karp theorem.
8. Integrality lemma**. Generalizations of flows. Menger's theorems about paths between pairs of points*.
9. Higher connectivity and edge-connectivity in graphs. Menger's related theorems*. The shortest path problem, conservative weighting. The Bellman-Ford algorithm*.

Theorems and statements with an * were partially proved in the lecture.
Theorems and statements with a ** were completely proved in the lecture.