Exercise-set 1.  
Solutions

1. (a) yes, yes,  
   (b) no, no,  
   (c) no, yes,  
   (d) no, yes.

2. 8 (the smallest possible and also realizable degree-sequence is 0,1,2,3,4,6).

3. 24.

4. Directed complete graph.

5. Only 18.

6. 6, 10, 15 or 30.

7. There are $n - 1$ odd degrees $\implies n - 1$ is even, $n$ is odd $\implies$ one even degree in $\overline{G}$.

8. The number of edges between the vertices of degree $\leq 7$ and the vertices of degree $\geq 16$ is both $\leq 70$ and $\geq 70 \implies 70 \implies |E(G)| = 115$.

9. No: the number of edges between the vertices of degree 4 and 10 and the vertices of degree 16 is both $\leq 66$ and $\geq 72$, a contradiction.

10. There are 11 of them.

11. There are 2 of them.

12. a) There are 4 such graphs,  
    b) there are 4 such graphs,  
    c) there are 2 such graphs.

13. a) Yes, yes, no (should have $15/2$ edges).  
    b) No (should have 11 vertices and $55/2$ edges).

14. a) no, b) yes.

15. a) and b) are isomorphic, and c) is not isomorphic to them.

16. a) and b) are isomorphic to it, c) is not.

17. No. The underlying graph is a cycle, and the knights cannot change their order on it.