1. When \( k \) committees have at least \( k \) members together, for \( k = 1, 2, \ldots \) (Hall’s condition).

2. a) Yes.
   b) No (\( H, J, L, M \) like only \( B, E, F \)).

3. (a) Count the number of edges between \( A \) and \( B \) in two ways.
   (b) Count the number of edges between \( X \) and \( N(X) \) in two ways.
   (c) Frobenius’ theorem.

4. Use Hall’s condition for (for the people-chocolates bipartite graph) for \(|X| \leq n \) and \(|X| \geq n + 1 \), resp.

5. Use Hall’s condition for \(|X| \leq \frac{n}{2} \) and \(|X| \geq \frac{n}{2} \), resp.

6. There is a non-connected counterexample.

7. Can select the edges greedily or use Hall’s condition.

8. a) Use Frobenius’ theorem.
    b) Use Hall’s theorem or unite the vertices of degree 3 and use exercise 3.

9. No perfect matching: \( N(\{a_1, a_2, a_4, a_6, a_8\}) = \{b_2, b_3, b_6, b_8\} \).

10. The (rows, columns; coins) bipartite graph is 4-regular.

11. Hall’s condition holds for the (figures, sets; containment) bipartite 4-regular (multi)graph.

12. First: \(|A| = |B| = 10\), then check Hall’s condition.

13. a) \( \nu(G) = \tau(G) = 8 \), a minimum covering set is \( \{a_2, a_3, a_6, a_8, b_1, b_4, b_7, b_9\} \).
    b) \( \nu(G) = \tau(G) = 8 \), a minimum covering set is \( \{a_1, a_3, a_6, a_9, b_1, b_3, b_6, b_8\} \).

14. \( \nu(G) = \tau(G) = 100 \), \( \rho(G) = 102 \), a maximum matching e.g. is \( \{a_i, b_{i+1}\}, \ i = 1, 2, \ldots, 100\} \).

15. a) \( \nu(G) = \tau(G) = 6 \).
    b) \( \nu(G) = \tau(G) = 6 \).
    c) \( \nu(G) = \tau(G) = 9 \).
    d) \( \nu(G) = \tau(G) = 6 \).

16. a) \( \nu(G) = \tau(G) = 4 \).
    b) \( \alpha(G) = 6 \).

17. \( \nu(G) = \tau(G) = 4 \), \( \rho(G) = 10 \).

18. a) The two endpoints of the edges in the matching can get the same color.
    b) \( \overline{G} \) is a regular bipartite graph \( \implies \omega(G) = 50 \) and also \( \overline{G} \) contains a perfect matching \( \implies \chi(G) = 50 \).