1. a) Vertices of the same color are independent.
    b) It is a good coloring, if each vertex in the complement of a maximum independent set of vertices gets a different color.

2. No (counterexample).

3. a) It is an interval graph (exists a representation of the vertices with intervals).
    b) It is not an interval graph, since \( \chi(G) \neq \omega(G) \).
    c) It is not an interval graph, since there is no representation of the vertices with intervals.
    d) It is an interval graph.
    e) It is not an interval graph (the 5-cycle cannot be represented with intervals).
    f) It is an interval graph.

4. It is an interval graph (exists a representation of the vertices with intervals).

5. \( \chi(G) = \omega(G) = 11 \) (=max. \# of intervals though a point)

6. \( \omega = 10 \) in the original graph. We can delete at most 2 vertices from a clique \( \implies \chi = \omega \geq 8 \).

7. \( \chi(G) = 3, \nu(G) = 9, \tau(G) = 12, \alpha(G) = 6, \rho(G) = 9 \).

8. a) \( \nu(G) = 4, \tau(G) = 4, \alpha(G) = 6, \rho(G) = 6 \).
    b) \( \nu(G) = 5, \tau(G) = 5, \alpha(G) = 7, \rho(G) = 7 \).
    a) \( \nu(G) = 4, \tau(G) = 4, \alpha(G) = 6, \rho(G) = 6 \).

9. \( G = K_{668} \cup K_{668,669} \implies \chi(G) = 668, \nu(G) = 334+668 = 1002, \tau(G) = 667+668 = 1335, \alpha(G) = 1+669 = 670, \rho(G) = 334 + 669 = 1003 \).

10. \( \nu(G) = 20 = \tau(G) \).

11. \( \alpha(G) = 86, \tau(G) = 14, \nu(G) = 14, \rho(G) = 86 \).

12. \( \nu(G) = 25, \alpha(G) = 75 \).

13. a) \( \{b, c, g, h\} \) (\( \nu(G) = 4 \)).
    b) \( \{b, d, f, h\} \) (\( \nu(G) = 4 \)).

14. \( \nu(G) = 4 = \tau(G) \).

15. a) By contradiction: otherwise the matching would not be maximum.
    b) Follows from a).
    c) Follows from b) and Gallai’s theorem.

16. a) True.
    b) False.
    c) No.

17. \( G \) contains a Hamilton cycle \( \implies \nu(G) \geq \lceil 2k + 1/2 \rceil = k \), and \( \nu(G) \leq (2k + 1)/2 = k \).

18. If we add the edge \{u, v\} to \( G \) then it contains a Hamilton cycle.

19. If we add two new vertices (connected to all the old ones) to \( G \) then the new graph contains a Hamilton cycle.

20. \( \det M \neq 0 \implies \exists \) a nonzero elementary product, corresponding to a perfect matching.

21. \( |E(G)| \leq \binom{20}{2} + 20 \cdot 80 = 1790 \), and this is possible (example).

22. \( \nu(G) = 10 = \tau(G) \).