Exercise-set 9.
Solutions

1. a) max $m(f) = 8$, min cut: $X = \{S, A, F\}$,
   b) max $m(f) = 20$, min cut: $X = \{S, A, B, C\}$,
   c) max $m(f) = 30$, min cut: $X = \{S, B, C, E\}$,
   d) max $m(f) = 17$, min cut: $X = \{S, B, C, D, E\}$,
   e) max $m(f) = 24$, min cut: $X = \{S, A, D, G\}$,
   f) max $m(f) = 21$, min cut: $X = \{S, D, F\}$,
   g) max $m(f) = 14$, min cut: $X = \{S, A, B, F, I\}$,
   h) max $m(f) = 24$, min cut: $X = \{S, B, D, E, F\}$.
2. The cut with $X = \{S, C, D, F\}$ has capacity 15.
3. No, not true. Either find the max flow (which is 20), or notice that the capacity of a cut cannot be 19 (all the capacities are divisible by 3 except for 5), and use the Ford-Fulkerson theorem.
4. The capacity of the cut is 19, max $m(f) = 18$, min cut: $X = \{S, A, B, G, H\}$.
5. a) max $m(f) = 21$, min cut: $X = \{S, A, F, G\}$,
   b) max $m(f) = 17$, min cut: $X = \{S, B, D, F, G\}$,
   c) max $m(f) = 24$, min cut: $X = \{S, A, C, F, G\}$.
6. max $m(f) = 20$, min cut: $X = \{S, D, E\}$.
7. max $m(f) = 22$, min cut: $X = \{S, D, E\}$.
8. Yes: e must be in the minimum cut.
9. True (we can use augmenting paths of smaller values).
10. The s,t-cut with $X = V \setminus \{t\}$ is a minimum s,t-cut.
11. The min s,w-cut has capacity at least 100.