Introduction to the Theory of Computing 2.

Exercise-set 4.
Solutions

1. Not possible; possible.
2. $|V(G)| = \binom{8}{2} = 28$, $\deg(v) = \binom{6}{2} = 15\ \forall v \in V(G) \implies$ no Euler-circuit.
3. $|V(G)| = 2^4 = 16$, $\deg(v) = \binom{4}{2} = 6\ \forall v \in V(G)$, but G is not connected $\implies$ no Euler-circuit.
4. Construct a graph G: $V(G) =$ digits $= \{0, 1, \ldots, 9\}$, and u and v are adjacent $\iff u+v \neq 9$. This graph contains an Euler-circuit ($\deg(v) = 8\ \forall v \in V(G)$, connected) $\iff \exists n$.
5. Construct a graph G: $V(G) =$ letters, and u and v are adjacent $\iff$ can stand next to each other. This graph contains an Euler-circuit ($\deg(v) = 30$ for vowels and $\deg(v) = 10$ for consonants, connected). Length of the longest sequence of letters = length of an Euler-circuit + 1 = $|E(G)|+1 = \binom{10}{2} + 10 \cdot 21 + 1 = 256$.
6. Construct a graph G: $V(G) =$ children, and u and v are adjacent $\iff$ not next to each other in the circle. This graph contains an Euler-circuit ($\deg(v) = 8\ \forall v \in V(G)$, connected). Most number of passes = length of an Euler-circuit = $|E(G)| = 40$.
7. There are 8 vertices of odd degree $\implies 8/2 - 1 = 3$ climb-ups are needed .
8. 4 vertices have odd degrees, and not all of them are adjacent $\implies$ 1 edge is enough.
9. Both endpoints must have odd degrees $\implies$ only $\{A, E\}$ is good.
10. Wall up the door between F and G. The throne-room is H.
11. There can be at most 2 components $\implies$ adding one edge can make it connected, and the degrees will be OK.
12. a) There is no Hamilton cycle: if we delete 4 vertices we get 5 components.
    There is a Hamilton path: draw.
    b) There is no Hamilton cycle: if we delete 2 vertices we get 3 components.
    There is a Hamilton path: draw.
    c) There is no Hamilton cycle: if we delete 4 vertices we get 5 components.
    There is a Hamilton path: draw.
13. a) Yes (draw); yes.
    b) No (delete 11 vertices); yes (draw).
14. a) No (delete the 9 vertices divisible by 3 or 5).
    b) No as well.
15. a) If we delete 2 vertices we get 3 components $\implies$ need to add at least 1 edge. That is enough (draw).
    b) If we delete 2 vertices we get 4 components $\implies$ need to add at least 2 edges. That is enough (draw).
16. If we delete 1 vertex (the center) we get 100 components $\implies$ need to add at least 99 edges. That is enough (path).
17. a) Construct a graph G: $V(G) =$ squares, and the edges are the possible moves of the horse. This graph contains no Hamilton path: if we delete the 4 middle vertices we get 6 components.
    b) Construct a graph G: $V(G) =$ squares, and the edges are the possible moves of the horse. This graph contains no Hamilton cycle: if we delete the 12 vertices we get 13 components.
18. a) Yes,
    b) yes.